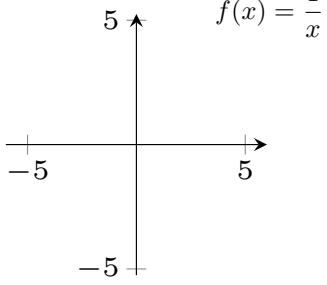
Objectives:

- Find limits where the variable goes to infinity and/or the limit is infinite.
- Find vertical and horizontal asymptotes of a given function.
- Choose and use an appropriate strategy to use with a given indeterminant form.

Limits Involving Infinity Graphically:

- $\lim_{x\to a^+} f(x) = \pm\infty$ and $\lim_{x\to a^-} f(x) = \pm\infty$ represent _____ .
- $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to-\infty} f(x) = L$ represent _____ .

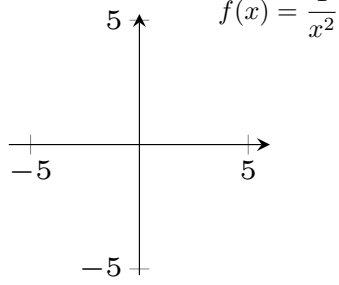
Common Functions with Limits Involving Infinity:



$f(x) = \frac{1}{x}$

$\lim_{x\to\infty} \frac{1}{x}$ $\lim_{x\to 0^+} \frac{1}{x}$

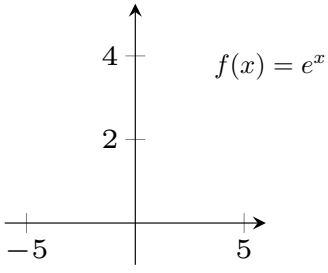
$\lim_{x\to-\infty} \frac{1}{x} =$ $\lim_{x\to 0^-} \frac{1}{x}$



$f(x) = \frac{1}{x^2}$

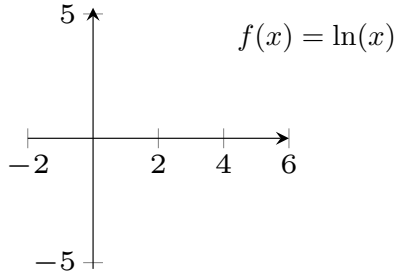
$\lim_{x\to\infty} \frac{1}{x^2}$ $\lim_{x\to 0^+} \frac{1}{x^2}$

$\lim_{x\to-\infty} \frac{1}{x^2}$ $\lim_{x\to 0^-} \frac{1}{x^2}$



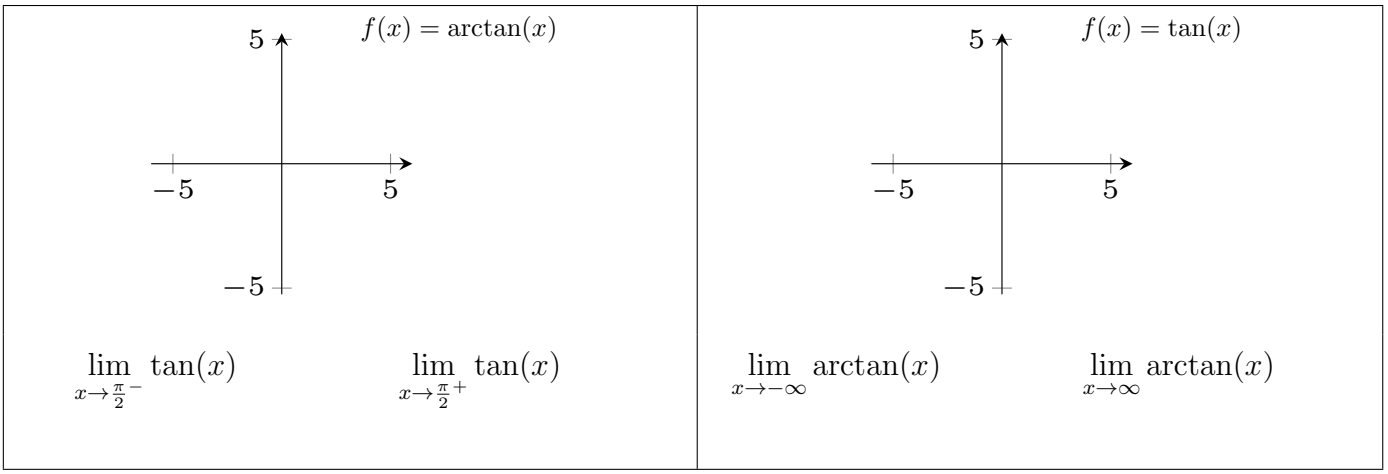
$f(x) = e^x$

$\lim_{x\to\infty} e^x$ $\lim_{x\to-\infty} e^x$



$f(x) = \ln(x)$

$\lim_{x\to\infty} \ln(x)$ $\lim_{x\to 0^-} \ln(x)$



Using These Common Functions:
Be careful with composite functions! Remember that the direction of the limit of the outside function depends on whether the inside function is increasing or decreasing.

1. $\lim_{x\to\infty} e^{\frac{1}{x}}$

2. $\lim_{x\to 0^+} e^{\frac{1}{x}}$

3. $\lim_{x\to 0^+} \ln(2^x)$

4. $\lim_{x\to\infty} \dfrac{1}{\ln x}$

5. $\lim_{x\to\infty} \ln\left(\dfrac{1}{x}\right)$

6. $\lim_{x\to\infty} \sin(\arctan x)$

Indeterminate Forms
Remember that we call the form "$\dfrac{0}{0}$" indeterminate. The forms _____ and _____ are also indeterminate. (We will see even more types of indeterminate forms later on.)

Useful Strategy:

Indeterminate Form Examples:

1. $\lim_{x\to\infty} \dfrac{2x^2+3}{x^2+x}$

2. $\lim_{x\to\infty} \dfrac{3x-1}{x^2+4}$

3. Find horizontal asymptotes of $f(x) = \dfrac{5x^2+7}{2x-4}$

4. $\lim_{x\to\infty} \dfrac{x}{\sqrt{x^2+1}}$

5. $\lim_{x\to\infty} (\sqrt{x^2+1}-x)$

6. $\lim_{x\to-\infty} (\sqrt{x^2+1}-x)$