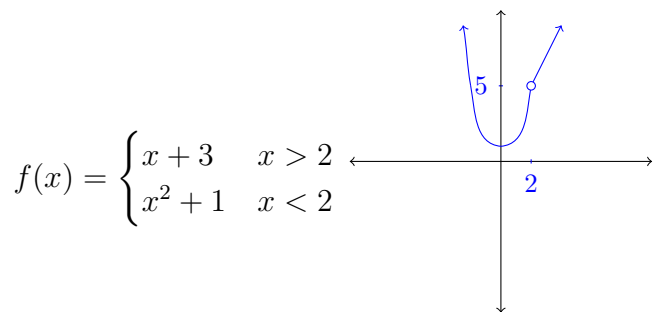


Objectives:

- Determine if a piecewise function is continuous
- State the Intermediate Value Theorem and use to determine information about a function

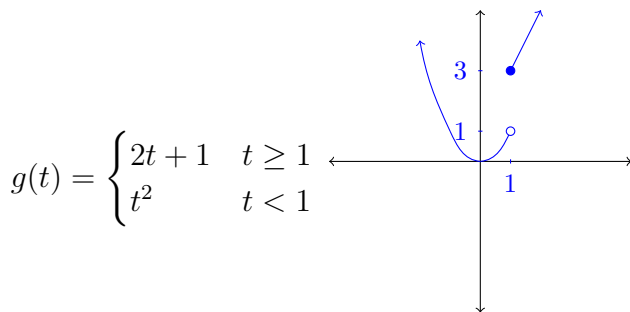
Continuity practice: Determine where each of these functions is continuous and classify the types of discontinuities. Sketch each graph.



$f(x)$ is defined piecewise with two polynomials and so $f(x)$ is continuous everywhere except possibly $x = 2$.

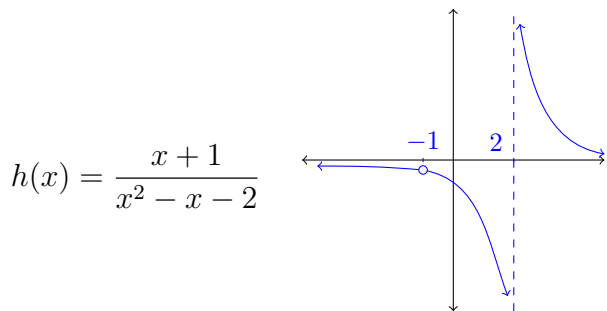
$f(2)$ is not even defined so $f(x)$ is discontinuous there.

$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (x^2 + 1) = 5$
 $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x + 3) = 5$
 So $\lim_{x \rightarrow 2} f(x)$ exists, so $f(x)$ has a removable discontinuity at $x = 2$.



$g(t)$ is defined piecewise with two polynomials and so $g(t)$ is continuous everywhere except possibly $t = 1$.

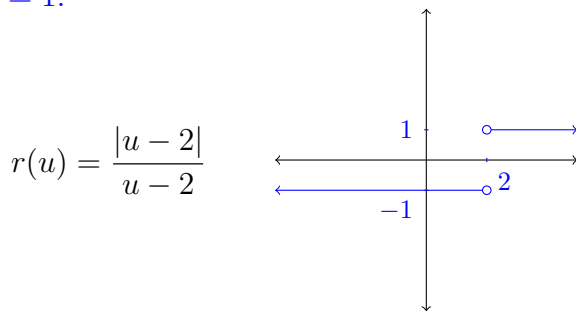
$\lim_{t \rightarrow 1^-} g(t) = \lim_{t \rightarrow 1^-} t^2 = 1$
 $\lim_{t \rightarrow 1^+} g(t) = \lim_{t \rightarrow 1^+} (2t + 1) = 3$
 So $\lim_{t \rightarrow 1} g(t)$ does not exist. But, since the right and left limits exist but differ, $g(t)$ has a jump discontinuity at $t = 2$.
 Since $\lim_{t \rightarrow 1^+} g(t) = g(1)$, $g(x)$ is right-continuous at $t = 1$.



$h(x)$ is a rational function so h is discontinuous only where undefined, at $x = -1$ and $x = 2$.

$\lim_{x \rightarrow -1} h(x) = \lim_{x \rightarrow -1} \frac{x+1}{(x+1)(x-2)} = \lim_{x \rightarrow -1} \frac{1}{(x-2)} = -\frac{1}{3}$
 So $h(x)$ has a removable discontinuity at $x = -1$.

$\lim_{x \rightarrow 2^+} h(x) = \lim_{x \rightarrow 2^+} \frac{1}{x-2} = \infty$ (think: “ $\frac{1}{\text{tiny pos.}}$ ”)
 $\lim_{x \rightarrow 2^-} h(x) = \lim_{x \rightarrow 2^-} \frac{1}{x-2} = -\infty$ (think: “ $\frac{1}{\text{tiny neg.}}$ ”)
 So $h(x)$ has an infinite discontinuity at $x = 2$ (a vertical asymptote).

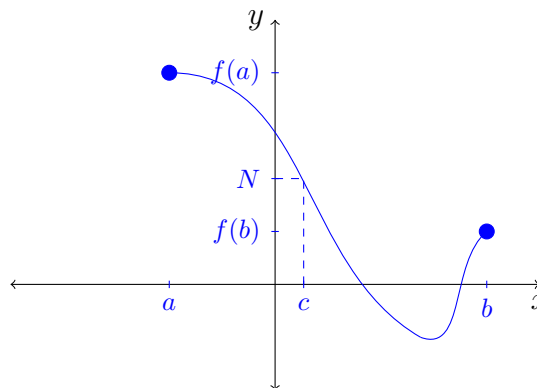


$\frac{|u - 2|}{u - 2} = \begin{cases} \frac{u-2}{u-2} = 1 & u > 2 \\ \frac{-(u-2)}{u-2} = -1 & u < 2 \end{cases}$
 $r(u)$ is undefined at $u = 2$ so r is discontinuous there.

$\lim_{u \rightarrow 2^+} r(u) = \lim_{u \rightarrow 2^+} 1 = 1$
 $\lim_{u \rightarrow 2^-} r(u) = \lim_{u \rightarrow 2^-} -1 = -1$
 $\lim_{u \rightarrow 2} r(u)$ DNE. But, since the right and left limits exist but differ, $r(u)$ has a jump discontinuity at $u = 2$.

Intermediate Value Theorem: Suppose f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then:

there exists a number c between a and b such that $f(c) = N$.

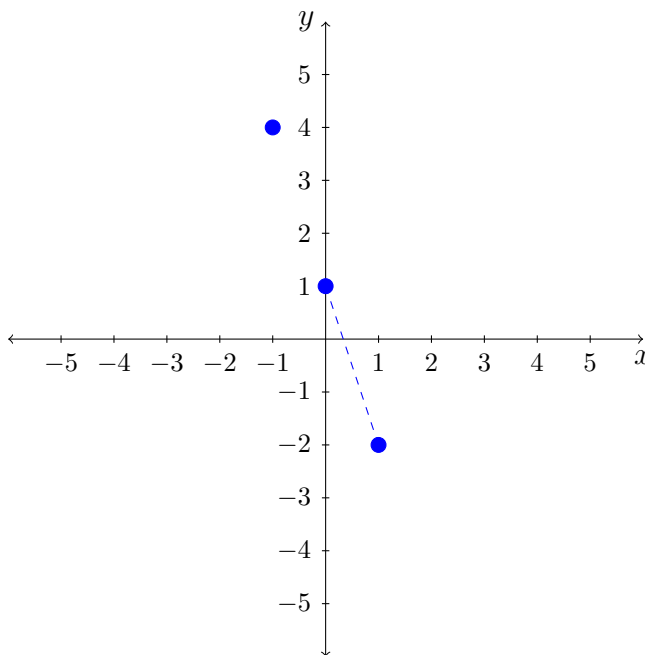


Example: Show that $f(x) = 3x^3 - 6x + 1$ crosses the x -axis somewhere.

Since $f(x)$ is a polynomial, $f(x)$ is continuous. So we can use the Intermediate Value Theorem.

$$\begin{aligned} f(-1) &= 4 \\ f(0) &= 1 \\ f(1) &= -2 \end{aligned}$$

Since 0 is between 1 and -2 , there exists some value c between $x = 0$ and $x = 1$ where $f(c) = 0$.



Note: We could also use the IVT to show (for example):

- $f(c) = \pi$ for some c between $[-1, 0]$.
- $f(x)$ also has roots in $[-2, -1]$ and $[1, 2]$.