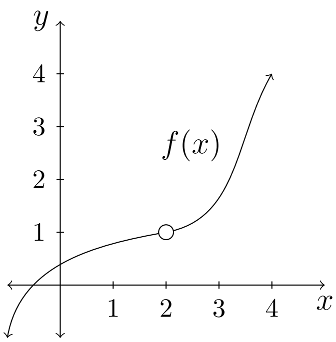
Goals:

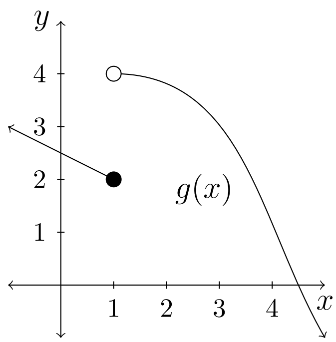
- Define limits of functions (one-sided and two-sided).
- Compute limits of functions graphically and numerically.

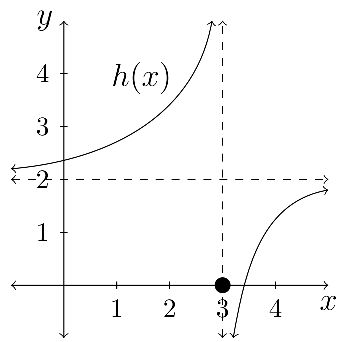
Intuition:

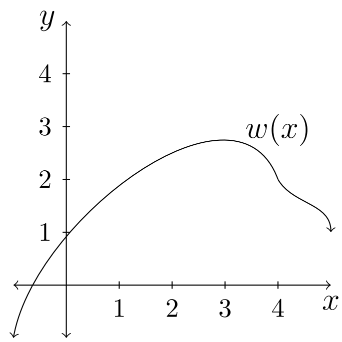
The limit of a function asks "what value is this function getting near to?" This is not always the same as the value of the function.

Graphically:









$\lim_{x\to 2} f(x) = 1$

$f(2)$ undefined

$\lim_{x\to 1^+} g(x) = 4$

$\lim_{x\to 1^-} g(x) = 2$

$\lim_{x\to 1} g(x)$ DNE

$g(1) = 2$

$\lim_{x\to 3^-} h(x)$ DNE, ∞

$\lim_{x\to 3^+} h(x)$ DNE, $-\infty$

$\lim_{x\to 3} h(x)$ DNE

$h(3) = 0$

$\lim_{x\to \infty} h(x) = 2$

$\lim_{x\to 4} w(x) = 2$

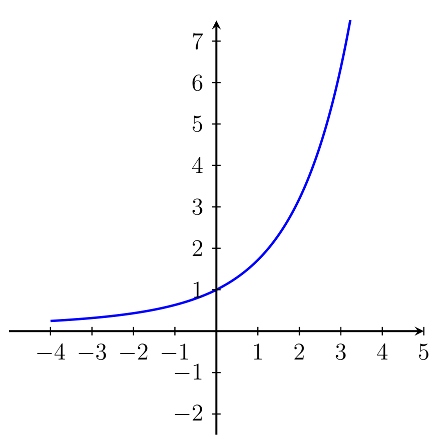
$w(4) = 2$

Numerically:

x	$\frac{e^x - 1}{x}$
0	undefined
0.01	≈ 1.00502
-0.01	≈ 0.99502
0.001	≈ 1.0005

Guess $\lim_{x\to 0} \frac{e^x - 1}{x}$:

1



Some Definitions:

- **LIMIT** If we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close to a (on either side of a) but not equal to a, we say "the limit of $f(x)$, as x approaches a, equals L" and we write,

$$\lim_{x \to a} f(x) = L.$$

- **RIGHT-HAND LIMIT** We say "the right-hand limit of $f(x)$, as x approaches a [or the limit of $f(x)$ as x approaches a from the right], equals L" and we write,

$$\lim_{x \to a^+} f(x) = L.$$

  if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close but not equal to a AND x greater than a (on the right-hand side of a)

- **LEFT-HAND LIMIT** We say "the left-hand limit of $f(x)$, as x approaches a [or the limit of $f(x)$ as x approaches a from the left], equals L" and we write,

$$\lim_{x \to a^-} f(x) = L.$$

  if we can make the values of $f(x)$ arbitrarily close to L by taking x to be sufficiently close but not equal to a AND x less than a (on the left-hand side of a)

FACT: For a function $f(x)$,

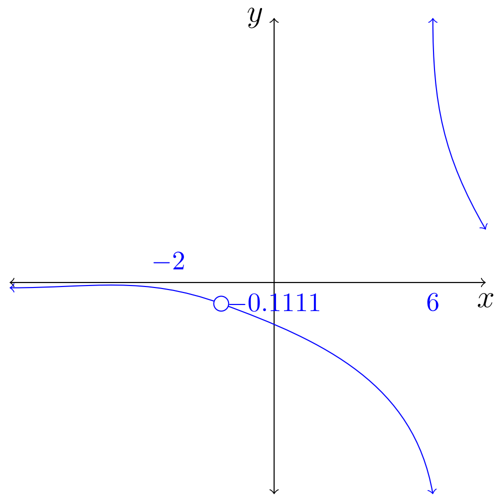
$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$.

Consider the function

$$f(x) = \frac{x+2}{x^2 - 5x - 14}.$$

Use at least five values of x to approximate $\lim_{x \to -2} f(x)$ and sketch the graph (including the scale).

x	$\frac{x+2}{x^2-5x-14}$
-1.9	-0.1124
-2.1	-0.1099
-1.95	-0.1117
-2.05	-0.1105
-1.99	-0.1112
-2.01	-0.111



Sketch a function $f(x)$ satisfying the following:

- $\lim_{x \to -\infty} f(x) = 0$
- $\lim_{x \to \infty} f(x) = \infty$
- $\lim_{x \to -1^-} f(x) = -2$
- $\lim_{x \to -1^+} f(x) = 2$
- $f(-1) = 0$
- $\lim_{x \to 1^+} f(x) = 2$
- $\lim_{x \to 1^-} f(x) = -1$