

1. Let X be a smooth, proper curve of genus $g > 0$ over a field k and let ω_X be its canonical sheaf. Show that a basis for $H^0(X, \omega_X)$ gives a map $f : X \rightarrow \mathbf{P}^{g-1}$. (In other words, show that ω_X has no base points.)
2. Show that if f is not a closed embedding then X is hyperelliptic. (Hint: Show that, if f is not a closed embedding then there are two points P and Q of X (possibly the same) such that $H^0(X, \omega_X(-P - Q)) = H^0(X, \omega_X(-P))$.)
3. Conclude that every genus 3 curve is either a plane quartic curve or is hyperelliptic.