

1. Suppose that X is a scheme of dimension 0 over a field k . Show that $\dim H^0(X, F) = \chi(X, F)$ for any coherent sheaf F on X .
2. Suppose that D and E are curves in \mathbf{P}_k^2 of degrees d and e , such that $D \cap E$ has dimension 0. Show that there is an exact sequence:

$$0 \rightarrow \mathcal{O}_{\mathbf{P}^2}(-d-e) \rightarrow \mathcal{O}_{\mathbf{P}^2}(-d) \oplus \mathcal{O}_{\mathbf{P}^2}(-e) \rightarrow \mathcal{O}_{\mathbf{P}^2} \rightarrow \mathcal{O}_{D \cap E} \rightarrow 0$$

3. Show that $\dim H^0(D \cap E, \mathcal{O}_{D \cap E}) = de$.