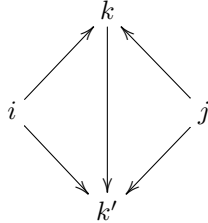


Suppose I is a category. A *cospan* from $i \in I$ to $j \in I$ is a diagram $i \leftarrow k \rightarrow j$. The cospans from i to j form a category, in which a morphism is a commutative diagram:



A diagram I is called *cosifted* if, for all i and j in I , the category of cospans from i to j is connected.

1. Prove that, if I is cosifted and $\{X_i\}_{i \in I}$ and $\{Y_i\}_{i \in I}$ are diagrams of sets then $\varinjlim (X_i \times Y_i) \rightarrow \varinjlim (X_i) \times \varinjlim (Y_i)$ is a bijection.
2. Give an example of a noncosifted colimit and finite product that do not commute.
3. Let \mathcal{A} be an abelian category and let $\mathbf{K}(\mathcal{A})$ be the category of chain complexes valued in \mathcal{A} . Show that for each $X_\bullet \in \mathbf{K}(\mathcal{A})$, the category of quasi-isomorphisms $X'_\bullet \rightarrow X_\bullet$ is cosifted.
4. Conclude that $\mathcal{D}(\mathcal{A})$ satisfies AB1 (it has a zero object, has finite products and coproducts, $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} : X \amalg Y \rightarrow X \times Y$ is an isomorphism for all X and Y) and that $\text{Ch}(\mathcal{A}) \rightarrow \mathcal{D}(\mathcal{A})$ is additive.
5. Suppose that $\text{Ch}(\mathcal{A}) \rightarrow \mathcal{C}$ is a functor that takes quasi-isomorphisms to isomorphisms. Show that if $f, g : X \rightarrow Y$ are chain homotopic maps in $\text{Ch}(\mathcal{A})$ then their images in \mathcal{C} are the same.