1. Show that if X is a paracompact topological space, Y is a topological space, and $\{Z_i\}$ is a locally finite closed cover of X then a function $f : X \to Y$ is continuous if and only if $f\big|_{Z_i}$ is continuous for every i.

2. Prove that a paracompact, Hausdorff topological space is normal.

3. Suppose that X is paracompact and Hausdorff, that F is a soft sheaf on X, and that

$$0 \to F \to G \to H \to 0$$

is exact. Show that

$$0 \to F(X) \to G(X) \to H(X) \to 0$$

is also exact. (Hint: given $\alpha \in H(X)$, begin by finding a locally finite closed cover $X = \bigcup_{i \in I} Z_i$ such that $\alpha\big|_{Z_i}$ lifts to $G(Z_i)$ for every i. Then find a maximal subcollection $J \subset I$ with a lift of $\alpha\big|_{\bigcup_{i \in J} Z_i}$. Finally, show that $\bigcup_{i \in J} Z_i = X$.)

4. Let X be a C^∞ manifold of dimension n. For each p, and each open U in X, let Ω^p be the sheaf of real-valued C^∞ p-forms on U.

   (a) Verify that Ω^p is a sheaf.

   (b) Show that Ω^p is acyclic.

   (c) Let $d : \Omega^p \to \Omega^{p+1}$ be the exterior derivative. Show that the complex

   $$0 \to \underline{\mathbf{R}} \to \underline{C}^\infty \to \Omega^1 \to \Omega^2 \to \cdots \to \Omega^n \to 0$$

   is exact. You can make use the vanishing of de Rham cohomology on contractible manifolds here. Or you can construct a chain homotopy: when U is an open subset of $\mathbf{R}^n$ with coordinates $x_1, \ldots, x_n$, let a denote antidifferentiation with respect to x_n. Define $s : \Omega^p(U) \to \Omega^{p-1}(U)$ by taking $s(f dx_{i_1} \wedge \cdots \wedge dx_{i_p})$ to be $a(f) dx_1 \wedge \cdots \wedge dx_{i_{p-1}}$ if $i_p = n$ and to be 0 if $i_p < n$.

   (d) Deduce that sheaf cohomology of X with coefficients in $\underline{\mathbf{R}}$ coincides with de Rham cohomology of X.