1. Suppose F and G are sheaves on X. We call a morphism $\varphi : F \to G$ an injection, surjection, or bijection if every morphism of stalks $\varphi_x : F_x \to G_x$ is injective, surjective, or bijective, respectively. Prove the following statements:

   (a) $\varphi : F \to G$ is a monomorphism if and only if it is an injection. (For φ to be a monomorphism means: if $\psi, \psi' : H \to F$ are two morphisms of sheaves then $\psi = \psi'$ if and only if $\varphi\psi = \varphi\psi'$.)

   (b) $\varphi : F \to G$ is an epimorphism if and only if it is a surjection. (For φ to be an epimorphism means: if $\psi, \psi' : G \to H$ are two morphisms of sheaves then $\psi = \psi'$ if and only if $\psi\varphi = \psi'\varphi$.)

   (c) $\varphi : F \to G$ is an isomorphism if and only if it is a bijection.

2. Complete the proof from class that the category of sheaves of abelian groups on a topological space X form an abelian category. (The main point is to show the existence of cokernels.)

3. (a) Show that arbirtrary small direct sums of sheaves exist by sheafifying the presheaf direct sum.

   (b) I claimed incorrectly in class that the presheaf direct sum of sheaves is always a sheaf. Give a counterexample.

   (c) Show that arbitrary small products of sheaves are sheaves (no sheafification required).

   (d) Show that $(\prod F_i)_x \neq \prod((F_i)_x)$.

   (e) Remark on which constructions behave well from the espace étalé point of view and which behave well for the functor point of view. (Hint: from the functor point of view, sheaves are characterized as respecting certain limits; from the espace étalé point of view, sheaves are characterized as colimits of certain objects.)

4. Complete the proof from class that the category of sheaves of abelian groups has enough injectives. (You need to show three things: existence of small direct sums, exactness of filtered colimits, and existence of a generator.)