

1. Adapt the proof from Wednesday's class to show the following: If  $h : X \rightarrow Y$  is a continuous function,  $F$  is a sheaf on  $X$ , and  $E$  is étale over  $Y$  then there is a natural bijection  $\text{Hom}(h^{-1}(F^{\text{ét}}), E) \simeq \text{Hom}(F, h_*(E^{\text{sh}}))$ .
2. Suppose that  $X$  is a topological space. Verify that a presheaf  $F$  on  $X$  is a sheaf if and only if the map

$$F(U) \rightarrow \varprojlim_i F(V_i)$$

is a bijection whenever  $\{V_i\}$  is a diagram of open subsets of  $U \in \text{Op}(X)$  and  $\varinjlim V_i = U$  in the category of topological spaces.

3. A constant presheaf is a presheaf of the form  $F(U) = S$  for a fixed set  $S$ . Why is a constant presheaf usually not a sheaf?
4. Show that the espace étalé of the constant presheaf with value  $S$  on  $X$  is  $X \times S$ .
5. A sheaf  $F$  on  $X$  is called locally constant if there is an open cover  $X = \bigcup U_i$  such that each  $F|_{U_i}$  is the sheafification of a constant presheaf. Show that  $F$  is a locally constant sheaf if and only if its espace étalé is a covering space of  $X$ .