1. Suppose that R is a noetherian ring and that there is an exact sequence

$$0 \to M' \to M \to M'' \to 0$$

of finitly generated R-modules. Show that if q is an associated prime of M then q is an associated prime of M' or of M''. (Hints: observe that if q is the annihilator of x in M then either q is the annihilator of x in M'' or there is some $y \in R - q$ such that $yx \in M'$.)

2. Show that if q is a prime ideal of R then the only associated prime of R/q is q.

3. Conclude that a finitely generated module over a noetherian ring R has only finitely many associated primes.

4. Here is a neat lemma that Mumford uses to calculate the cohomology of line bundles on abelian varieties: suppose that R is a regular local ring of dimension g and

$$0 \to K^0 \to K^1 \to \cdots \to K^{g-1} \to K^g \to 0$$

is a complex of finitely generated, free R-modules. If the $H^i(K^\bullet)$ are *artinian* (this means that the maximal ideal p of R acts nilpotently) then $H^i(K^\bullet) = 0$ for all $i \neq g$. (Hint: choose a regular element x of R and consider the $R/(x)$-complex $K^\bullet/xK^\bullet$.) You can find a proof of this lemma on p. 127 of Mumford's book on abelian varieties.