

1. Suppose that R is a commutative ring, S is a multiplicatively closed subset of R , and P is a projective R -module. Show that $S^{-1}P$ is a projective $S^{-1}R$ -module.
2. Conclude that, if R is noetherian, and R has finite global dimension, then so does $S^{-1}R$.
3. If M is finitely generated and R is noetherian, show that $S^{-1}\text{Hom}_R(M, N) \simeq \text{Hom}_{S^{-1}R}(S^{-1}M, S^{-1}N)$. Conclude that $S^{-1}\text{Ext}_R^n(M, N) \simeq \text{Ext}_{S^{-1}R}^n(S^{-1}M, S^{-1}N)$.
4. Conclude that if R is noetherian and I is an injective R -module then $S^{-1}I$ is an injective, both as an $S^{-1}R$ -module and as an R -module.