

1. Let $A = \mathbf{C}[x, y]$ and let $B = A/(x, y)$. Compute $\text{Ext}^n(B, A)$ and $\text{Ext}^n(B, B)$ for all n .
2. Let $A = \mathbf{C}[x, y]/(xy)$ and let $B = A/(x, y)$. Compute $\text{Ext}^n(B, A)$ and $\text{Ext}^n(B, B)$ for all n .
3. Let $A = \mathbf{C}[x]/(x^2)$ and let $B = \mathbf{C}[x]/(x)$. Compute $\text{Ext}^n(B, A)$ and $\text{Ext}^n(B, B)$ for all n .
4. Let $\hat{\mathbf{Q}} = \varprojlim \mathbf{Q}/n\mathbf{Z}$. Prove that $\hat{\mathbf{Q}} \simeq \mathbf{R}$:
 - (a) Show that $\hat{\mathbf{Q}}$ is divisible.
 - (b) Show that $\hat{\mathbf{Q}}$ is torsion-free.
 - (c) Conclude that $\hat{\mathbf{Q}}$ is a \mathbf{Q} -vector space.
 - (d) Show that $\hat{\mathbf{Q}}$ has the cardinality of \mathbf{R} .
 - (e) Conclude that $\hat{\mathbf{Q}}$ has a basis whose cardinality is the same as the cardinality of \mathbf{R} . (You may use the axiom of choice.)
 - (f) Conclude that $\hat{\mathbf{Q}} \simeq \mathbf{R}$.
5. Complete the verification that Yoneda extensions $\mathbf{Y}. \text{Ext}(X, Y)$ (Yoneda extensions) has the structure of an abelian group.
6. Complete the verification that $\text{Hom}(X, -)$ and $\mathbf{Y}. \text{Ext}(X, -)$ together form a truncated δ -functor. Why is the sequence constructed in class exact?