1. Let R be a commutative ring and let S be a multiplicatively closed subset of R. Form a diagram J whose objects are the elements of S, with $\mathrm{Hom}(x, y) = \{z \in S \mid zx = y\}$. Show that J is a filtered diagram.

2. If M is an R-module and $S \subset R$ is multiplicatively closed, construct a diagram $F : J \to R$-Mod where $F(x) = M$ for all x and $F(u) : M \to M$ is multiplication by u whenever $u : x \to y$ is a map in J. Define $S^{-1}M = \varinjlim F$. Show that $S^{-1}(-)$ is an exact functor.

3. Compute the kernel of $M \to S^{-1}M$.

4. Verify that if
$$0 \to Y \to Z \to X \to 0$$
is an exact sequence in an abelian category, and $X' \to X$ is any map, then the sequence
$$0 \to Y \to Z \underset{X}{\times} X' \to X' \to 0$$
is exact. (First make sure you know what the maps in the sequence are!)

5. Verify that if
$$0 \to Y \to Z \to X \to 0$$
is an exact sequence in an abelian category, and $Y \to Y'$ is any map, then the sequence
$$0 \to Y' \to Z \underset{Y}{\sqcup} Y' \to X \to 0$$
is exact. (Again, first make sure you know what the maps are.)