1. Prove that $\varinjlim$ is right exact. Show, in other words, that if C is an abelian category, D is a category, and there is an exact sequence of functors

$$X' \to X \to X'' \to 0$$

then the sequence

$$\varinjlim X' \to \varinjlim X \to \varinjlim X'' \to 0$$

is exact.

2. Suppose that R is a commutative ring and X and Y are diagrams of R-modules. Show that there is a canonical isomorphism

$$\varinjlim (X \otimes_R Y) \simeq (\varinjlim X) \otimes_R (\varinjlim Y).$$

3. Compute the colimit of the diagram of nonzero multiplication maps $\mathbf{Z} \to \mathbf{Z}$. To be precise, there is one object $X(a) = \mathbf{Z}$ of the diagram for each nonzero integer a and one map $a \to b$ whenever $a|b$. The map $\mathbf{Z} = X(a) \to X(b) = \mathbf{Z}$ is multiplication by b/a.

4. The opposite-category analogue of the colimit is the limit. Compute the limit of the diagram

$$\cdots \mathbf{Z} \xrightarrow{p} \mathbf{Z} \xrightarrow{p} \mathbf{Z}$$

where p is a prime number.

5. The limit of the diagram of all the quotient maps $\mathbf{Z}/p^n\mathbf{Z} \to \mathbf{Z}/p^m\mathbf{Z}$ (for $n \geq m$) is called $\hat{\mathbf{Z}}_p$. The limit of all of the quotient maps $\mathbf{Z}/m\mathbf{Z} \to \mathbf{Z}/n\mathbf{Z}$ (where $n|m$) is called $\hat{\mathbf{Z}}$. Show that $\hat{\mathbf{Z}} = \prod_p \hat{\mathbf{Z}}_p$ where $\hat{\mathbf{Z}}_p$ is the group of p-adic integers.

6. Show that $\varinjlim_n \varprojlim_m p^{-n}\mathbf{Z}/p^m\mathbf{Z} \simeq \varprojlim_m \varinjlim_n p^{-n}\mathbf{Z}/p^m\mathbf{Z} = \hat{\mathbf{Q}}_p$ is the group of p-adic rationals.

7. What is the difference between the rings $\mathbf{Q} \otimes \prod_p \hat{\mathbf{Z}}_p$ and $\prod_q \hat{\mathbf{Q}}_p$? The first one is called the ring of *finite adèles*.

8. Compute $\mathrm{Hom}(\mathbf{Q}, \mathbf{Q}/\mathbf{Z})$.

9. Do exercise 3.1.1.

10. Do exercise 3.1.2.

11. Do exercise 3.1.3.

12. Do exercise 3.2.2.

13. Do exercise 3.2.3.

14. Let R be a commutative ring and let S be a multiplicative subset of R. Show that $S^{-1}R$ is a flat R-module. (Hint: you may want to identify $S^{-1}R \otimes_R M$ with $S^{-1}M$ and show that $S^{-1}(-)$ is an exact functor.)