1. Our proof from class that there are enough injective R-modules uses several properties of R-modules that do not hold in every abelian category:

   (a) R-modules admit arbitrary (i.e., indexed by any small set) direct sums;

   (b) filtered colimits of R-modules are exact (i.e., if $\bigcup M_\beta = M_\alpha$ then $\bigcup f^{-1} M_\beta = f^{-1} M_\alpha$); and

   (c) R-modules has a generator (i.e., an object X such that $Y = 0$ if and only if $\mathrm{Hom}(X, Y) = 0$).

   Adapt the argument to show that any abelian category satisfying these properties has enough injectives. (This is Theorem 1.10.1 of Grothendieck's Tohoku paper.)

2. In class, we showed that we would get the same derived functors for tensor product if we derived with respect to the first variable or with respect to the second variable. In this exercise, you will prove this another way. For now, we take $\mathrm{Tor}_n(M, N)$ to be the derived functor of tensor product with respect to the *first* varaible.

   (a) Show that $(-) \otimes P$ is exact when P is projective. (This was done in class.)

   (b) Use 2a to show that $\mathrm{Tor}_*(M, P) = 0$ for all $n > 0$ when P is projective.

   (c) Use 2a to show that $\mathrm{Tor}_*(M, -)$ is a δ-functor.

   (d) Use 2b to show that $\mathrm{Tor}_*(M, -)$ is an *effaceable* δ-functor.

   (e) Observe that $\mathrm{Tor}_*(M, -)$ and $\mathrm{Tor}_*(-, M)$ are both universal δ-functors and conclude that the isomorphism $M \otimes N \simeq N \otimes M$ extends to isomorphisms $\mathrm{Tor}_n(M, N) \simeq \mathrm{Tor}_n(N, M)$.

3. Do exercise 2.7.5.

4. Do exercise 2.7.3.