

1. Show that an object  $P$  is projective if and only if  $\text{Hom}(P, -)$  is exact.
2. Show that  $P$  is projective if and only if every epimorphism  $Q \rightarrow P$  is split.
3. Prove that the ideal  $(2, 1 + \sqrt{-5}) \subset \mathbf{Z}[\sqrt{-5}]$  is projective. Can you show it is not free?
4. Prove that the ideal  $(x, y) \subset \mathbf{C}[x, y]/(y^2 - x^3 - x)$  is projective. Can you show it is not free?
5. Let  $M$  be the  $\mathbf{C}[x, y]$ -module  $\mathbf{C}$ , on which  $x$  and  $y$  both act by 0. Compute  $\text{Tor}_n(M, M)$  for all integers  $n$ .
6. Prove that if  $A$  and  $B$  are abelian groups,  $\text{Tor}_n(A, B) = 0$  for all  $n \neq 0, 1$ .
7. Do exercise 2.2.3 from Weibel.
8. Exercise 2.2.1 from Weibel is interesting but not essential.