1. Complete the verification of the universal property of the cone from class. Show, in other words, that if $f : K_\bullet \to L_\bullet$ is a morphism of chain complexes, $\mathrm{cone}(f)$ is the universal example of a chain complex equipped with a map $g : L_\bullet \to \mathrm{cone}(f)$ and a *specified* chain homotopy $s : gf \simeq 0$.

2. Exercise 1.5.3 is very similar in spirit to the last one, except it is about the mapping cylinder.

3. Complete the verification from class that, for any $f : K_\bullet \to L_\bullet$, there is a canonical chain homotopy equivalence $\mathrm{cone}(\mathrm{cone}(f)) \simeq K_\bullet[-1]$.

4. Check that the map $H_i(K_\bullet) \to H_i(L_\bullet)$ induced from the short exact sequence
$$0 \to L_\bullet \to \mathrm{cone}(f) \to K_\bullet[-1] \to 0$$
is the same as the one induced from f.

5. This problem requires some definitions:

   Definition 1. A morphism $f : A \to B$ in a category $\mathcal{C}$ is called a *monomorphism* if $f \circ (-) : \mathrm{Hom}(Z, A) \to \mathrm{Hom}(Z, B)$ is injective for all $Z \in \mathcal{C}$.

   A *split monormorphism* is a map $f : A \to B$ such that there exists a map $g : B \to A$ such that $gf = \mathrm{id}_A$.

   (a) Verify that split monomorphisms are monomorphisms.

   (b) Check that if $f : A \to B$ is a monomorphism in an abelian category then $B \simeq A \oplus B/A$.

   (c) Show that for any $f : A \to B$ in an abelian category, the map $\ker(f) \to A$ is a monomorphism.

   (d) Suppose that $f : K_\bullet \to L_\bullet$ is a monomorphism in the chain homotopy category and that the chain homotopy category is abelian. Show that the map $\mathrm{cone}(f)[1] \to K_\bullet$ must be chain homotopic to zero.

   (e) Show that a chain homotopy between the map $\mathrm{cone}(f)[1] \to K_\bullet$ and zero gives a splitting of the map $K_\bullet \to L_\bullet$.

   (f) Conclude that every monomorphism in the chain homotopy category is a split monomorphism.

   (g) Conclude that if a map $f : K_\bullet \to L_\bullet$ has a kernel in the chain homotopy category then $\ker(f)$ is a direct summand of $K_\bullet$.

   (h) Show that the multiplication-by-2 map $[2] : \mathbf{Z}[0] \to \mathbf{Z}[0]$ does not have a kernel in the chain homotopy category. (Recall that we proved in class that the kernel is not 0.)