1. In class we considered a complex

$$K_m^{(n)} = \mathbf{Z}^{\binom{n+2}{m+1}}.$$

$K_m^{(n)}$ has a basis consisting of the $(m+1)$-element subsets $\{i_0 < \cdots < i_m\}$ of $\{0, \ldots, n+1\}$. The boundary map $d : K_m^{(n)} \to K_{m-1}^{(n)}$ is given by the formula

$$d\{i_0 < \cdots < i_m\} = \sum (-1)^j \{i_0 < \cdots < \hat{\imath}_j < \cdots < i_m\}.$$

We constructed several exact sequences:

$$0 \to \mathbf{Z}[1] \to K_\bullet^{(n)} \to C_\bullet(\Delta^{n+1}) \to 0 \quad (1)$$

$$0 \to C_\bullet(\partial\Delta^{n+1}) \to C_\bullet(\Delta^{n+1}) \to \mathbf{Z}[-(n+1)] \to 0 \quad (2)$$

$$0 \to K_\bullet^{(n-1)} \to K_\bullet^{(n)} \to K_\bullet^{(n-1)}[-1] \to 0 \quad (3)$$

(a) Compute the homology of $K_\bullet^{(-1)}$.

(b) Use the long exact sequence associated with (3) and induction to compute the homology of $K_\bullet^{(n)}$ for every n.

(c) Use the long exact sequence associated with (2) to compute the homology of $C_\bullet(\Delta^{(n+1)})$.

(d) Use the long exact sequence associated with (1) to compute the homology of $C_\bullet(\partial\Delta^{(n+1)})$. Be careful with the case $n = 0$.

2. Prove that the connecting morphism in the snake lemma is natural with respect to the diagram and is uniquely determined by this naturality. (Hint: if you look at the proof from class carefully, you'll see that we constructed the connecting homomorphism using its naturality.)