1. Prove that addition of homomorphisms is associative and bilinear in abelian categories.

2. Show that
$$0 \to A \xrightarrow{f} B \xrightarrow{g} C$$
is exact if and only if f factors through an isomorphism $A \simeq \ker(g)$.

3. Show that
$$A \xrightarrow{f} B \xrightarrow{g} C \to 0$$
is exact if and only if g factors through an isomorphism $\mathrm{coker}(f) \simeq C$.

4. Show that
$$0 \to A \xrightarrow{f} B \to 0$$
is exact if and only if f is an isomorphism.

5. Suppose that
$$X \xrightarrow{f} A \xrightarrow{g} Y$$
are homomorphisms in an abelian category and $gf = 0$. Show that
$$(Y : A)/X \simeq Y : (A/X).$$

6. Show that
$$A \xrightarrow{f} B \xrightarrow{g} C$$
is exact if and only if $gf = 0$ and $C : B/A = 0$.

7. Show that the sequence
$$A \to B \xrightarrow{f} C \to D$$
is exact if and only if f induces an isomorphism
$$B/A \simeq D : C.$$

8. Suppose that
$$A \to B \to C \to D$$
is exact and there is a map $X \to B$. Show that
$$A \to B/X \to C/X \to D$$
is also exact.