1. Suppose $\mathcal{C}$ is a category with a zero object (an object that is both initial and final). Show that, for an $f : a \to b$ in $\mathcal{C}$, there is a unique map $\operatorname{coim}(f) \to \operatorname{im}(f)$ such that

$$a \to \operatorname{coim}(f) \to \operatorname{im}(f) \to b$$

coincides with f.

2. If you know something about Lie groups: show that the category of commutative Lie groups is not abelian. Which axiom fails?

3. Suppose $\mathcal{C}$ is an abelian category. A short exact sequence in $\mathcal{C}$ is a sequence of maps

$$0 \to A \to B \to C \to 0$$

such that $A \to B$ is the kernel of $B \to C$ and $B \to C$ is the cokernel of $A \to B$. A morphism of short exact sequences is a commutative diagram:

$$\begin{array}{ccccccccc} 0 & \longrightarrow & A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & A' & \longrightarrow & B' & \longrightarrow & C' & \longrightarrow & 0 \end{array}$$

What are the initial and final objects, the products and coproducts, the kernels and cokernels, and the images and coimages in this category? (Try to describe these as explicitly as possible.) Is this category abelian?