Math 6210: Algebraic topology

Jonathan Wise

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1 Office hours and contact information

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Office hours (provisional!)	MWF, 3–4pm

2 Prerequisites

The prerequisites for this class are fairly few. The main one is to be comfortable writing proofs. I will also expect you to be familiar with the metric topology on the real numbers—that is, with the notions of open and closed set, and their relationships to distance and covergent sequences. Transfinite induction plays an important role in topology, and will appear at least once early in the semester; I will review the concept when it comes up, as well as some of its other incarnations (the well-order principle, Zorn's lemma), but it will be useful if you already have some familiarity with it. In later parts of the course certain concepts from algebra will become important: notably, the theory of groups—both abelian and non-abelian—and vector spaces will play important roles in homotopy theory and homology; I will assume you have seen these structures before but will review the more unfamiliar concepts and constructions when they arise.

3 Textbook

The primary textbook for this class is

> J. Munkres. *Topology*, second edition. Prentice Hall. ISBN-10: 0131816292. ISBN-13: 978-0131816299.

I will also use

> J. Hocking and G. Young. *Topology*. Dover. ISBN-10: 0486656764. ISBN-13: 978-0486656762.

The outline of the course will most closely follow the Hocking and Young text, although I recommend studying from Munkres' book.

Here is another excellent, but more advanced, topology text:

A. Hatcher. *Algebraic Topology.* Cambridge University Press. ISBN 0-521-79540-0. Also available online: www.math.cornell.edu/∼hatcher/AT/AT.pdf.

I may use this text as a reference for some of the latter topics in the class.

4 Syllabus

The following are the main topics of this course. We will not have time to cover all of them in great depth, so please if you have particular topics you would like to see, please share your preferences with me.

- Point set topology. Our basic notion of space is a set equipped with a *topology*, which is extra information to indicate which points are near to which others. Our first examples of topological spaces will be metric spaces—sets equipped with a notion of distance between points. We will cover some of the fundamental properties of topological spaces: connectedness, compactness, and the various separation axioms (especially the "Hausdorff" axiom). We will also discuss some of the key constructions of new topological spaces: products, quotients, gluing, and function spaces. The big theorems in this part of the course are Tychonoff's theorem, Urysohn's lemma, the Arzelà–Ascoli theorem, and the Baire category theorem.

- Homotopy theory. Homotopy refers to continuous deformation and is a fundamental notion in algebraic topology. We will learn about the fundamental group of a space (homotopy classes of maps from the circle into that space) and the higher homotopy groups (homotopy classes of maps from higher dimensional spheres). The computation of homotopy groups is one of the most difficult problems in algebraic topology and one that has driven many of its advances. We will discuss techniques for computing the fundamental group (the van Kampen theorem essentially solves this problem) and compute fundamental groups of a number of spaces. If time and interest permit, we will investigate a few higher homotopy groups. However, the main techniques for the higher homotopy groups come from homology (see the Hurewicz theorem, which will unfortunately be beyond the scope of this course). We will also cover the classification of covering spaces and local systems. One neat application is the classification of subgroups of free groups. The fundamental group can also be used to prove some famous results like the Jordan curve theorem, the fundamental theorem of algebra, Brouwer fixed point theorem (in 2 dimensions).[1]

- Homology. Homology is a system of abelian groups associated to any topological space. Although homology groups are more difficult to define than homotopy groups, they are much easier to compute.[2] The main tools for calculation are the Mayer–Vietoris theorem and excision. We will do many calculations and study a few of the abundant applications.

- Cohomology. We probably will not cover cohomology in this course, except perhaps for brief discussion at the end. The theory is dual to homology and serves many of the same purposes, but has several more pleasant features: for one, cohomology is a ring

[1] However, much easier—and in some cases, more general—proofs are possible using homology.

[2] The reason for this is that the van Kampen theorem for fundamental groups does not generalize to higher homotopy groups, but the Mayer–Vietoris theorem applies to all homology groups.

whereas homology is merely an abelian group; cohomology is also generalizes more easily than homology in some abstract settings.

5 Exams and grading

There will be two take-home exams: one midterm and one final.

Here is a grading breakdown:

40%	Homework	
30%	Midterm	due Oct. 24
40%	Final	due Dec. 18

These dates are provisional and may be adjusted.

6 Homework

Homework will be due on alternating Wednesdays, starting on 9/5. Some problems will be required and others will be marked as optional. You will be required to complete a certain number of the optional problems over the course of the semester. The idea here is that the optional problems will generally be about applications of the topics considered in class and you will be able to select applications that conform with your interests.

Unless I specify otherwise, you are free—and encouraged—to work together and to use outside resources on your assignments. However, **you must cite any resources you use**, including both texts you consulted (this also refers to resources on the internet) and people with whom you discussed the assignment. If you work with others on an assignment, **be sure to write up your solutions alone**: this way you will get all of the benefits of collaboration but also ensure that you understand every step of what is written on your paper. Unless you are explicitly permitted to turn in assignments as a group, **do not write up solutions while discussing the problems with others**: this invariably leads to situations where a student submits a solution that he does not completely understand himself.

I recommend approaching homework assignments in the following sort of way: First, spend some time thinking about the problems yourself; identify the parts that seem easy and those that seem hard. Then discuss the problems with one to three other students. Destroy your notes when you end your discussion and then go home to write your solutions independently.

You will find that solutions to many of the problems on your assignments can be found online or in the library. However, you will get more out of your homework if you make a genuine effort to solve the problems yourself before consulting outside resources. That said, you should also balance the impulse to solve every problem yourself against your time constraints: if you are really stuck on a problem, seek out a hint—from me, from a classmate, from a textbook, or from the internet—and keep trying. And remember to cite the hint when you turn in your assignment.

7 Conflicts, issues, rules and regulations

If any conflicts or other issues come up, or you feel uncomfortable in class for any reason—academic or otherwise—please let me know. I will do what I can to resolve the issue. Some of the things that come up most frequently are conflicts due to religious observance, accommodations for disabilities, and requests to be addressed by a different name or pronoun. I hope you will feel comfortable coming to me if these or any other issues arise.

The earlier you come to me, the easier it will be for me to help. I won't ask for specific information about your request unless it is necessary.

You can read more about the official university policies on these and related topics here:

www.colorado.edu/ememoarc/faculty/2012.08/0002.html

Of course, those policies will be respected in this course. Please note that in order to provide special accommodations for a disability I will need to have a letter from disability services explaining what accommodations are appropriate. This letter may take time to procure, so please contact Disability Services early if you may qualify for special accommodations.