1. Compute the number of ways of coloring the faces of an icosahedron with 3 colors, with two colorings considered equivalent if one can be transformed to the other by a rigid symmetry.

2. Let X be the tesseract (4-dimensional hypercube). The vertices of X are the points $(\pm 1, \pm 1, \pm 1, \pm 1)$ in $\mathbb{R}^4$. Compute the size of the group of rigid symmetries of X. (Rigid symmetries of X are always given by matrices that take vertices to vertices, have determinant 1, and satisfy $M^T M = 1$.)

3. Let X be a set with 6 elements. Let W be the set of 2-element subsets of X. Compute the size of W.

   (a) A syntheme is a partition of X into 3 disjoint subsets with 2 elements in each part. In other words, a syntheme is a 3-element subset $\{w_1, w_2, w_3\}$ of W such that $\bigcup w_i = X$. Let Y be the set of synthemes of X. Calculate the size of Y.

   (b) A *pentad* is a collection of 5 synthemes $y_1, \ldots, y_5 \in Y$ such that $\bigcup y_i = W$. Equivalently, $y_i \cap y_j = \varnothing$ for $i \neq j$. Let Z be the set of pentads of X. Calculate the size of Z.

   (c) Show that S_X acts on Z in a natural way. Obtain a homomorphism $S_X \to S_Z$ and show that it is bijective. (Hint: It is probably easiest to show it is surjective and that both groups have the same size.)

   (d) Choose a bijection $Z \simeq X$ and obtain an automorphism of S_X. Show that this is not an inner automorphism. (Hint: What happens to cycle types under this automorphism?)

4. Prove that the group of rigid symmetries (= rotational symmetries) of the icosahedron is isomorphic to A_5. (Hint: Find a 5-element set to act on.)