1. Suppose that G is a group and $H \subset G \times G$ is an equivalence relation on the underlying set of G. Let G/H be the set of equivalence classes of H. Prove that H is a subgroup of $G \times G$ if and only if there is a group structure on G/H such that $G \to G/H$ is a homomorphism.

2. Fix a positive integer n and consider the set

$$X = \{\pm \prod_{1 \leq i < j \leq n} (x_i - x_j)\}$$

where the x_i are indeterminates. (So X consists of two polynomial functions.) For example, if $n = 3$ then $X = \{(x_1 - x_2)(x_1 - x_3)(x_2 - x_3), -(x_1 - x_2)(x_1 - x_3)(x_2 - x_3)\}$. Show that the formula $g.f(x_1, \ldots, x_n) = f(x_{g(1)}, \ldots, x_{g(n)})$ gives an action of S_n on X. Conclude that S_n has a subgroup of index 2, called the *alternating group* and denoted A_n.