I will show that every group of size 90 has a nontrivial normal subgroup. Suppose that G is a group with $|G| = 90$.

First consider the 5-Sylow subgroups. The number of these is $\equiv 1 \bmod 5$ and divides $90/5 = 18$, hence is either 1 or 6. If the number is 1 then the unique 5-Sylow is normal, so assume the number is 6. Then there are $6 \times (5-1) = 24$ elements of G of order 5.

Now consider the 3-Sylows. Each has order 9 and the number is $\equiv 1 \bmod 3$ and divides $90/9 = 10$, hence is either 1 or 10. If the number is 1 then the unique 3-Sylow is normal, so we can assume that there are ten 3-Sylow subgroups.

I claim that at least two of the 3-Sylow subgroups must intersect nontrivially. Indeed, if not then the number of elements of G of order divisible by 3 is $10 \times (9-1) = 80$ but there isn't enough room for 90 elements among the $90 - 24 = 66$ elements of G not of order 5.

Now suppose that P and Q are 3-Sylow subgroups with nontrivial intersection. Then $|P \cap Q| = 3$ since the size cannot be 1 or 9 and must divide 9. Then $|PQ| = \frac{|P|\,|Q|}{|P \cap Q|} = 27$ so the subgroup $\langle P, Q \rangle \subset G$ must have size ≥ 27. On the other hand, the size of $\langle P, Q \rangle$ is divisible by 9 and divides 90. The only numbers satisfying these criteria are 45 and 90. In the first case $\langle P, Q \rangle$ has index 2, hence is normal in G. In the second case, $\langle P, Q \rangle = G$. But both P and Q have order 9, which is the square of a prime, so P and Q are both abelian. Therefore $P \cap Q$ is normal in both P and Q, hence is normal in $\langle P, Q \rangle = G$.

(The idea to consider the group generated by two 3-Sylows came from here: `http://math.stackexchange.com/a/360973/89750`.)