Math 6130 Fall 2015
Exam 1

October 29, 2015

1. Prove that every finite group G is isomorphic to a subgroup of $\mathrm{GL}_n(\mathbb{C})$ for some positive integer n.

2. Classify all groups of order $253 = 11 \times 23$ up to isomorphism.

3. (a) Prove that if p is a prime and H is a subgroup of G then the number of p-Sylow subgroups of H is at most equal to the number of p-Sylow subgroups of G.

    (b) Nilpotent groups are defined inductively: The trivial group is nilpotent and a group G is said to be *nilpotent* if $Z(G)$ is nontrivial and $G/Z(G)$ is nilpotent. Prove that every Sylow subgroup of a nilpotent group is normal. Conclude that every nilpotent group is the product of its Sylow subgroups.