Math 52 – Spring 2012

Assignment #5

Problem 1. *§14.2, #16*

Problem 2. *§14.1, #11–14*

Problem 3. *§14.1, #20*

Problem 4. *§14.1, #36 (here* **r** *is the vector field whose value at the point with coordinates* (x, y, z) *is the vector* (x, y, z)*)*

Problem 5. *Demonstrate the following identities:*

(a) If f is a nice function (meaning you can take arbitrarily many derivatives of it) in 3 *variables, then* $\text{curl}(\text{grad} f) = 0$*.*

(b) If F is a vector field in 3 *dimensions whose components are nice functions (in the same sense), then* $\text{div}(\text{curl}(F)) = 0$*.*

Problem 6. *Which of the following vector fields are gradients of* differentiable *functions? Give the function or give a reason why the function is not a gradient.*

(a) $F(x, y) = (2x, 3y)$

(b) $F(x, y, z) = (y^3, y^2, y)$

(c) $F(x, y) = (e^x y, e^x)$

(d) (extra credit) $F(x, y) = \left(\dfrac{-y}{x^2 + y^2}, \dfrac{x}{x^2 + y^2}\right)$

Could the following function be the curl of a vector field:

(e) $F(x, y, z) = (y^3, y^2, y)$*?*

Problem 7. *Suppose that T is a coordinate transformation from* $\mathbf{R}^2$ *(with coordinates u and v) to a surface S in* $\mathbf{R}^3$ *(with coordinates x, y, and z) such that*

(i) $T(u, 0) = (u^2, 3u + 2, 1)$ *and*

(ii) $T(0, v) = (v^3 - v^2, 2, v + 1)$.

Compute $\left|J_T(0, 0)\right|$.

Problem 8. *We now have two different coordinate systems on (the surface of) a sphere of radius* 1*:*

(i) in the spherical coordinate system*, the coordinates are* (θ, ϕ)*, which correspond to the point on the sphere with*

$$x = \cos(\theta)\sin(\phi) \qquad y = \sin(\theta)\sin(\phi) \qquad z = \cos(\phi);$$

(ii) *in the* stereographic coordinate system, *the coordinates are* (u, v), *which correspond to the point on the sphere with*

$$x = \frac{2u}{u^2 + v^2 + 1} \qquad y = \frac{2v}{u^2 + v^2 + 1} \qquad z = \frac{u^2 + v^2 - 1}{u^2 + v^2 + 1}.$$

(a) *Find a formula for the stereographic coordinates in terms of* (x, y, z). *(This is the inverse of the transformation* T *you found in Problem 6 of last week's homework.)*

(b) *Suppose that a point of the sphere has spherical coordiantes* (θ, ϕ). *Find its stereographic coordinates.*

(c) *The line* $v = 0$ *corresponds to a curve* C *on the surface of the sphere. Describe this curve and draw a sketch of it on the surface of the sphere.*

(d) *The line* $u + v = 1$ *corresponds to another curve* C' *on the surface of the sphere. Sketch this one too.*

(e) *Transform the curve* C *into spherical coordinates and draw the corresponding curve on axes labelled* θ *and* ϕ. *(Hint: this curve will have several pieces.)*

(f) *Find the spherical coordinates of the intersection points of* C *and* C'. *(Hint: there are two such points.)*