Math 52 – Spring 2012

Assignment #4

Problem 1. *§13.9, #8*

Problem 2. *§13.9, #12*

Problem 3. *§9.5, #12*

Problem 4. *§13.8, #12 (check your answer using Pappus's theorem)*

Problem 5. *Let R be the segment of the parabola $y = x^2$ with $\left|x\right| \leq a$. Express your answers in terms of 1-variable integrals; you don't have to evaluate those integrals.*

(a) Set up an integral to find the length of R.

(b) Find the centroid of R.

Problem 6. *Let S be the sphere with equation $x^2 + y^2 + z^2 = 1$ (just the surface of the sphere, not the interior). The stereographic coordinate system on S associates to each pair of real numbers (u, v) a unique point on S, according to the following rule: $T(u, v)$ is the unique point on S that also lies on the line connecting $(0, 0, 1)$ to $(u, v, 0)$. In this problem you will find a formula for $T(u, v)$.*

(a) Observe that the line connecting two points $P = (x_1, y_1, z_1)$ and $Q = (x_2, y_2, z_2)$ consists of all points of the form

$$(1-s)P + sQ = \big((1-s)x_1 + sx_2, (1-s)y_1 + sy_2, (1-s)z_1 + sz_2\big)$$

where s is allowed to be any real number.

Your answer to this question should explain two things: why this is a line, and why it passees through the points P and Q.

(b) Use the last part to give an equation for the line L connecting $P = (0, 0, 1)$ and $Q = (u, v, 0)$.

(c) Solve for the nonzero value of s where the line you wrote down in the last part intersects the sphere S. Why aren't we interested in the point where $s = 0$?

(d) Determine the (x, y, z)-coordinates of the point on L corresponding to the value of s you computed in the last part.

(e) Write a formula for $T(u, v)$.

Problem 7. *(a) Find the centroid of the hemisphere defined by the equation $x^2 + y^2 + z^2 = b^2$ and $z \geq 0$. (Hint: use cylindrical coordinates.)*

(b) Use the 4-dimensional analogue of Pappus's theorem to compute the "boundary volume" of the 4-sphere of radius b (obtained by rotating the hemisphere from the last part around the xy-plane).