Math 52 – Spring 2012

Assignment #2

In this problem set, you can use a symbolic integrator to do 1-variable integrals. There is one (that I sometimes use) available online at Wolfram Alpha.

Problem 1. *§13.3, #6*

Problem 2. *§13.6, #12. You don't have to do a sketch.*

Problem 3. *§13.6, #34. In this problem, "centroid" refers to the center of mass. Suggestion: use symmetry to save some work.*

Problem 4. *§13.3, #30*

Problem 5. *§13.5, #24. Note that "centroid" means center of mass here.*

Problem 6. *§13.5, #42. You may want to check your answer with a direct calculation (see problem #41).*

Problem 7. *§13.6, #48*

Problem 8. *Areas and volumes of solid spheres. Fix a positive number r (the radius).*

(a) *Compute the centroid of a the line segment $0 \leq x \leq r$. Use this and Pappus's theorem to compute the area of a circle of radius r.*

(b) *Compute the centroid of the semicircle with $x^2 + y^2 \leq r^2$ and $0 \leq y \leq r$ and use Pappus's theorem to compute the volume of a sphere.*

(c) *Compute the centroid of the hemisphere with $x^2 + y^2 + z^2 \leq r^2$ and $0 \leq z \leq r$ and then use (a generalization of) Pappus's theorem to compute the volume of a 4-dimensional sphere.*

The integrals in this problem can be done by hand, but they get pretty tricky to do without using polar coordinates (which we will cover next week). If you want to do the integrals by hand, you'll probably want to do a trigonometric substitution and use the half-angle identity $\cos(\theta)^2 = \frac{1+\cos(2\theta)}{2}$ more than once.

Problem 9. *Suppose that the plane is covered with equally spaced horizontal and vertical lines. The distance between adjacent horizontal lines is twice the diameter of a penny; the distance between adjacent vertical lines is three times the diameter of a penny.*

In all of the problems below, first set up an integral before calculating. Actually calculating the integral shouldn't require taking an antiderivative.

(a) *Compute the probability that a penny dropped on the plane will cross a line.*

(b) *Compute the probability that a penny dropped on the plane will cross a horizontal line* but not a vertical line.

(c) *Compute the expected (average) numbers of (i) horizontal lines, (ii) vertical lines, and (iii) lines of all types that the penny will cross.*

Problem 10. *People arrive at a party at uniformly distributed times between 5pm and 6pm. Compute*

(a) *the expected amount of time that the first guest must wait until the second guest arrives (assuming there is a total of 2 guests), and*

(b) *the expected amount of time that the first guest must wait until the third guest arrives (assuming there is a total of 3 guests).*