Math 52 – Spring 2012

Assignment #1

Problem 1. *Let* $f(x,y) = xy$ *and let* $R = [0,2] \times [0,2]$*. Let* P *be the partition of* R *into four squares having length* 1 *on each side.*

(a) Draw a picture of R *and* P*. In each of the four pieces of the partition* P*, label the points where* f *takes its smallest and largest values. (It may make your answer easier to read if you avoid overlapping labels by drawing separate pictures of the pieces of* P*.)*

(b) Compute the upper and lower bounds for $\int_R f\,dA$ *associated to Riemann sums with this partition.*

Problem 2. §*13.1, #18*

Problem 3. §*13.1, #30*

Problem 4. §*13.6, #2*

Problem 5. §*13.2, #6*

Problem 6. §*13.2, #18*

Problem 7. *Evaluate the following iterated integral:*

$$\int_{-1}^{0} \int_{-\sqrt{y+1}}^{0} \sqrt{\frac{x^3}{3} - x}\, dx\, dy.$$

For a hint (that may or may not actually be helpful), see the first few lines of the Wikipedia entry on elliptic integrals.

Problem 8. *Let* R *be the region bounded by the lines* $y = x + 1$*,* $y = 0$*, and* $y = 1 - x$*. Express* $\int_R f\,dA$

(a) as an iterated integral of the form $\iint f(x,y)\,dx\,dy$*, and*

(b) as an iterated integral of the form $\iint f(x,y)\,dy\,dx$*.*

(Your job is to supply the limits of the integrals.) It is okay to give your answer(s) as a sum of iterated integrals.

Problem 9. §*13.6, #10*

Problem 10. *Let* R *be the region in the plane described by the inequalities*

(i) $x^2 + y^2 \leq 4$

(ii) $x \geq 0$*, and*

(iii) $y \geq 0$*.*

Let $f(x, y) = x^2 + y^2$.

(a) *Draw a picture of* R.

(b) *Draw a picture of* R *subdivided into the following four regions:*

   (i) S_1 *consists of all points in* R *such that* $0 \leq \theta \leq \frac{\pi}{8}$ *in polar coordinates,*

   (ii) S_2 *consists of all points in* R *such that* $\frac{\pi}{8} \leq \theta \leq \frac{\pi}{4}$ *in polar coordinates,*

   (iii) S_3 *consists of all points in* R *such that* $\frac{\pi}{4} \leq \theta \leq \frac{3\pi}{8}$ *in polar coordinates,*

   (iv) S_4 *consists of all points in* R *such that* $\frac{3\pi}{8} \leq \theta \leq \frac{\pi}{2}$ *in polar coordinates.*

   Compute the upper and lower bounds for $\int_R f(x, y)\, dA$ *associated to this partition of* R.

(c) *Draw a picture of* R *subdivided into the following four regions:*

   (i) T_1 *consists of all points in* R *such that* $0 \leq r \leq \frac{1}{2}$ *in polar coordinates,*

   (ii) T_2 *consists of all points in* R *such that* $\frac{1}{2} \leq r \leq 1$ *in polar coordinates,*

   (iii) T_3 *consists of all points in* R *such that* $1 \leq r \leq \frac{3}{2}$ *in polar coordinates, and*

   (iv) T_4 *consists of all points in* R *such that* $\frac{3}{2} \leq r \leq 2$ *in polar coordinates.*

   Compute the upper and lower bounds for $\int_R f(x, y)\, dA$ *associated to this partition of* R.

(d) *Which partition gives a better estimate of* $\int_R f(x, y) dA$*? Give a conceptual reason why one partition works better than the other.*

(e) *(extra credit) Evaluate the integral* $\int_R f\, dA$ *exactly.*