Please choose a topic before the beginning of Thanksgiving break (11/16). I have to approve your topic, so please discuss it with me before the 16th, either in person or by e-mail.

Submit a draft by the end of Thanksgiving break (11/26). The final draft will be due one week later (12/3).

Possible paper topics are below. Feel free to suggest your own.

1. The fundamental group. [Mun]
2. Symmetries of fields and Galois theory.
3. Frieze patterns and their symmetry groups. [Fra]
4. Change ringing and generating S_n by transpositions.
5. The braid group.
6. Tiling of regions. [Thu]
7. Symmetry groups in Escher.
8. What lengths can be constructed using straightedge and compass?
9. Topological groups (and Lie groups).
10. Symmetries of platonic solids.
11. Uniqueness and non-uniqueness of factorization into primes.
12. The fundamental theorem of arithmetic.
13. The p-adics.
14. Mathematical card tricks (or other mathematical magic tricks). [DG]
15. Which integers are sums of squares? [Tan]
16. Algebraic groups. [Tan]
17. Rubik's cube. [LG, Lar]
18. The group law of an elliptic curve. [PT]
19. The Lorentz groups—symmetries of infinitesimal relativistic spacetime.
20. $\mathrm{SL}_2(\mathbf{Z})$: the 2×2 integer matrices with determinant 1, an extremely important and beautiful group.
21. The quaternions: a 4-dimensional analogue of the complex numbers. [Wae]
22. Symmetries of crystals (e.g., arising in physics or chemistry).
23. The structure of finitely generated abelian groups.

References

[DG] Persi Diaconis and Ron Graham. *Magical mathematics*. Princeton University Press, Princeton, NJ, 2012. The mathematical ideas that animate great magic tricks, With a foreword by Martin Gardner.

[Fra] John B. Fraleigh. *A first course in abstract algebra*. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., seventh edition edition, 2002. ISBN-10: 0201763907, ISBN-13: 978-0201763904.

[Lar] Mogens Esrom Larsen. Rubik's revenge: the group theoretical solution. *Amer. Math. Monthly*, 92(6):381–390, 1985.

[LG] Jing Li and Melissa Gymrek. The mathematics of the rubik's cube. web.mit.edu/sp268/www/rubik.pdf.

[Mun] James R. Munkres. *Topology: a first course*. Prentice-Hall Inc., Englewood Cliffs, N.J., 1975.

[PT] Paris Pamfilos and Apostolos Thoma. Apollonian cubics: an application of group theory to a problem in Euclidean geometry. *Math. Mag.*, 72(5):356–366, 1999.

[Tan] Lin Tan. The group of rational points on the unit circle. *Math. Mag.*, 69(3):163–171, 1996.

[Thu] William P. Thurston. Conway's tiling groups. *Amer. Math. Monthly*, 97(8):757–773, 1990.

[Wae] B. L. van der Waerden. Hamilton's discovery of quaternions. *Mathematics Magazine*, 49(5):pp. 227–234, 1976.