Math 3140 — Fall 2012
Assignment #6

Due Monday, Oct. 15. Remember to cite any references you use.

Exercise 38. Let Y be the set $\{1, 2, 3\}$ and let X be the set of all subsets of Y.

(a) How many elements does X have?

(b) How many elements does S_X have?

(c) For each $\sigma \in S_3$, let $T_\sigma : X \to X$ be the function

$$T_\sigma(A) = \big\{\sigma(x) \bigm| x \in A\big\}.$$

Show that for each $\sigma \in S_3$ the function T_σ is in S_X.

(d) Let $\varphi : S_3 \to S_X$ be the function given by $\varphi(\sigma) = T_\sigma$. Show that φ is a homomorphism.

(e) Draw a picture with one vertex for every element of X and a line connecting two vertices A and B if there is a $\sigma \in S_3$ such that $T_\sigma(A) = B$.

(f) For each $A \in X$, let

$$O_A = \big\{B \in X \bigm| \exists \sigma \in S_3,\ T_\sigma(A) = B\big\}.$$

This is called the **orbit** of A under the action of S_3. A subset of X is called an orbit of S_3 if it is the orbit of some element of X under the action of S_3. Compute the size of each orbit of S_3.

(g) For each $A \in X$, let G_A be the set of $\sigma \in S_3$ such that $T_\sigma(A) = A$. Compute G_A for each $A \in X$ and compute its size.

(h) Conjecture a relationship between the size of G_A and the size of O_A.