Math 3140 — Fall 2012
Assignment #5

Due Friday, Oct. 12. Solutions may be submitted early for comments.

Write a careful solution to each of the problems below, using complete sentences and omitting no steps. Work alone. Do not consult classmates, the help lab, or anyone else. Textual references are allowed, but make sure to document any you use.

Exercise 1. Let $\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/m\mathbf{Z}$ be the group whose elements are pairs (a, b) with $a \in \mathbf{Z}/n\mathbf{Z}$ and $b \in \mathbf{Z}/m\mathbf{Z}$. The group operation is

$$(a, b) + (a', b') = (a + a', b + b').$$

Prove that n and m are *relatively prime* integers **if and only if**[1] $\leftarrow_1$

$$\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/m\mathbf{Z} \cong \mathbf{Z}/mn\mathbf{Z}.$$

You do not have to verify that $\mathbf{Z}/n\mathbf{Z} \times \mathbf{Z}/m\mathbf{Z}$ is a group.

Exercise 2. Prove that a homomorphism of groups $\varphi : G \to H$ is injective if and only if $\ker(\varphi) = \{1\}$ (where 1 is the identity of G). (Hint: if $\varphi(x) = \varphi(y)$, what is $\varphi(xy^{-1})$?)

[1] updated wording!