Math 3140 — Fall 2012
Handout #15

Exercise 1. Find a field with 25 elements.

Exercise 2. Is there is a field with 6 elements?

Exercise 3. Find a field with 27 elements.

Exercise 4. Let F be a field with 4 elements.

(a) Find a degree 2 polynomial in $F[x]$ that is irreducible.

(b) Find a field with 16 elements.

Exercise 5. (a) Find all irreducible polynomials of degree 2 in $\mathbf{F}_3[x]$.

(b) For each polynomial P you found above there is a field $\mathbf{F}_3[x]/(P)$. Which of these are isomorphic to each other?

(c) How many elements do these fields have?

Exercise 6. [**?**, §23, #9]

(a) Factor the polynomial $x^4 - 1$ into irreducible polynomials over $\mathbf{F}_5$.

(b) Find a ring isomorphism between $\mathbf{F}_5[x]/(x^4 - 1)$ and $\mathbf{F}_5 \times \mathbf{F}_5 \times \mathbf{F}_5 \times \mathbf{F}_5$.

Exercise 7. For which $\lambda \in \mathbf{F}_{13}$ is $\mathbf{F}_{13}[x]/(x^3 - \lambda)$ a field?

Exercise 8. (a) Find all elements $\alpha \in \mathbf{R} \times \mathbf{C}$ such that $\alpha^3 = 1$.

(b) Find all homomorphisms $\varphi : \mathbf{R}[x]/(x^3 - 1) \to \mathbf{C}$ such that $\varphi(\alpha) = \alpha$ for all $\alpha \in \mathbf{R}$.

(c) Show that $\mathbf{R}[x]/(x^3 - 1)$ is isomorphic to $\mathbf{R} \times \mathbf{C}$.