Math 3140 — Fall 2012
Handout #13

Exercise 1. Each of the rings in the first list below is isomorphic to one of the rings in the second list. Determine which ones and construct the isomorphisms.

(a) $\mathbf{R}[x]/(x^2-1)$

(b) $\mathbf{R}[x]/(x^2+1)$

(c) $\mathbf{R}[x]/(x^2)$

(i) the subring of $M_2(\mathbf{R})$ consisting of all matrices of the form $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$ with $a, b \in \mathbf{R}$

(ii) the subring of $M_2(\mathbf{R})$ consisting of all matrices of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ with $a, b \in \mathbf{R}$

(iii) the subring of $M_2(\mathbf{R})$ consisting of all matrices of the form $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$ with $a, b \in \mathbf{R}$

Exercise 2. (a) For what values of λ is $\mathbf{R}[x]/(x^2-\lambda)$ isomorphic to $\mathbf{R} \times \mathbf{R}$?

(b) For what values of λ is $\mathbf{R}[x]/(x^2-\lambda)$ isomorphic to $\mathbf{C}$?

Exercise 3. (a) Is $\mathbf{Q}[x]/(x^2-4)$ isomorphic to $\mathbf{Z}[y]/(y^2-1)$?

(b) Is $\mathbf{Q}[x]/(x^2-3)$ isomorphic to $\mathbf{Z}[y]/(y^2-1)$?

(c) For which values of λ is $\mathbf{Q}[x]/(x^2-\lambda)$ isomorphic to $\mathbf{Q}[y]/(y^2-1)$?

Exercise 4. Suppose that λ and μ are integers such that $\mathbf{Q}[x]/(x^2-\lambda)$ and $\mathbf{Q}[x]/(x^2-\mu)$ are isomorphic to each other. What does this tell us about λ and μ? (Hint: what elements of $\mathbf{Q}[x]/(x^2-\lambda)$ are squares?)

Exercise 5. Find an isomorphism between $\mathbf{R}[x]/(x^3-1)$ and a familiar ring.