Math 3140 — Fall 2012
Handout #12

Definition 1. Let A and B be rings. A **homomorphism** from A to B is a function

$$\varphi : A \to B$$

satisfying the following two properties:

(i) $\varphi(x + y) = \varphi(x) + \varphi(y)$ and

(ii) $\varphi(xy) = \varphi(x)\varphi(y)$.

A homomorphism is called an **isomorphism** if it is bijective.

Exercise 1. For each pair of rings below, give a reason why the two rings are not isomorphic to each other:

(1) $\mathbf{Z}/n\mathbf{Z}$, $n \neq 0$.

(2) $\mathbf{Z}$

(3) $\mathbf{Q}$

(4) $\mathbf{R}$

(5) $\mathbf{C}$

(6) the Gaussian integers, $\mathbf{Z}[i]$

(7) the ring A consisting of all symbols $a + bx$ with $a, b \in \mathbf{R}$ and

$$(a + bx) + (a' + b'x) = (a + a') + (b + b')x$$
$$(a + bx)(a' + b'x) = (aa' + bb') + (ab' + a'b)x,$$

(8) the ring B consisting of all symbols $a + b\epsilon$ with $a, b \in \mathbf{R}$ and

$$(a + b\epsilon) + (a' + b'\epsilon) = (a + a') + (b + b')\epsilon$$
$$(a + b\epsilon)(a' + b'\epsilon) = aa' + (ab' + a'b)\epsilon,$$

(9) the ring of $M_2(\mathbf{R})$ of all 2×2 matrices with real coefficients,

(10) the ring $\mathbf{R}[x]$ of polynomials in one variable with real coefficients.

Exercise 2. Find all ring homomorphisms from $\mathbf{Z}[i]$ into itself.