Let $\alpha_1, \alpha_2, \alpha_3$ be complex numbers. Consider the expression

$$\delta = (\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_3).$$

Suppose that the polynomial

$$(x - \alpha_1)(x - \alpha_2)(x - \alpha_3)$$

distributes to

$$x^3 + ax^2 + bx + c.$$

There is a formula

$$\delta^2 = f(a, b, c)$$

where f is a polynomial. Find f. (Hint: it might be helpful to try the problem in the case where $a = 0$ first.)