Question 1. Prove the following statements about linear transformations:

(a) If $f : U \to V$ and $g : V \to W$ and $h : V \to W$ are linear transformations then $(g + h) \circ f = g \circ f + h \circ f$.

(b) If $f : U \to V$ and $g : U \to V$ and $h : V \to W$ are linear transformations then $h \circ (f + g) = h \circ f + h \circ g$.

(c) If $f : U \to V$ and $g : V \to W$ and $h : W \to X$ are linear transformations then $h \circ (g \circ f) = (h \circ g) \circ f$.

(d) If $f : U \to V$ is a linear transformation and $\mathrm{id}_U : U \to U$ and $\mathrm{id}_V : V \to V$ are the identity transformations then $\mathrm{id}_V \circ f = f \circ \mathrm{id}_U = f$.

(e) If λ is a scalar and $f : U \to V$ and $g : V \to W$ are linear transformations then $\lambda.(g \circ f) = (\lambda.g) \circ f = g \circ (\lambda.f)$.

Question 2. Prove the following statements about matrices:

(a) If F is an $n \times p$ matrix and G and H are $m \times n$ matrices then $(G + H)F = GF + HF$.

(b) If F and G are $n \times p$ matrices and H is an $m \times n$ matrix then $H(F + G) = HF + HG$.

(c) If F is an $p \times q$ matrix and g is an $n \times p$ matrix and H is an $m \times n$ matrix then $H(GF) = (HG)F$.

(d) If F is an $m \times n$ matrix and I_m is the $m \times m$ identity matrix and I_n is the $n \times n$ identity matrix then $I_m F = F I_n = F$.

(e) If λ is a scalar and F is an $n \times p$ matrix and G is an $m \times n$ matrix then $\lambda.(GF) = (\lambda.G)F = G(\lambda.F)$.