

Question 1. Is there a linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $T(1, 1, 1) = (1, 1)$ and $T(2, 2, 2) = (1, 2)$?

Question 2. Suppose that V is a 1-dimensional vector space over a field F . Describe all linear transformations from V to V . (Hint: suppose that $T : V \rightarrow V$ is a linear transformation and $\{\mathbf{v}\}$ is a basis of V . What is $T(\mathbf{v})$? If \mathbf{w} is any vector in V , can you use $T(\mathbf{v})$ to figure out $T(\mathbf{w})$?)

Question 3. Let V be the vector space of linear plane motions and let $T : V \rightarrow V$ be the transformation ‘reflection through the dashed line’. Indicate $T(\mathbf{v})$ for each of the vectors provided.

Question 4. Let V be the vector space of linear plane motions and let $T : V \rightarrow V$ be the transformation ‘rotation by the angle indicated’. Draw $T(\mathbf{v})$ for each of the vectors \mathbf{v} provided.

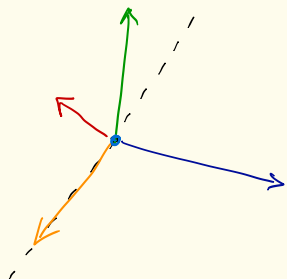
Question 5. Let V be the vector space of linear plane motions and let $T : V \rightarrow V$ be the transformation ‘projection onto the dashed line’. Draw $T(\mathbf{v})$ for each of the vectors \mathbf{v} provided.

Question 6. Let V be the vector space of linear plane motions and let $T : V \rightarrow V$ be the shear parallel to the dashed line. Draw $T(\mathbf{v})$ for each of the vectors \mathbf{v} provided.

Question 7. Let V be the vector space of linear plane motions and let $T_i : V \rightarrow V$ be the transformation ‘projection onto the line labelled i ’. Let T be the linear transformation $2T_1 + \frac{1}{2}T_2$. Draw $T(\mathbf{v})$ for each of the vectors \mathbf{v} provided.

Question 8. Find a way to construct the reflection and shear transformations as linear combinations of projections and the identity transformation.

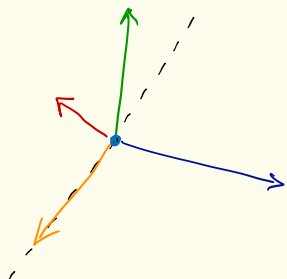
Reflection (#3)



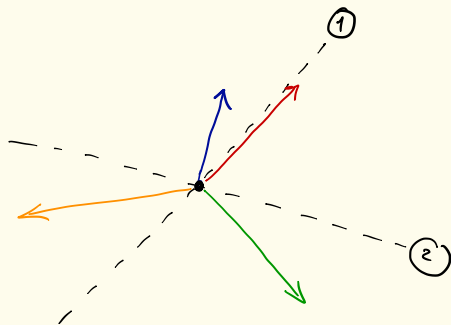
Rotation (#4)



Projection (#5)



Directional scaling (#7)



Shear (#6)

