Graded Problem Set #1

Math 2135 Spring 2020

Due Friday, January 24

1. Express each of the following vectors as a single column vector whose entries are real numbers:

   (i)
   $$\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 1 \\ -3 \end{pmatrix}$$

   (ii)
   $$\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

2. Solve the following equation for a vector $\mathbf{x}$ in $\mathbb{R}^2$:
   $$\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}\mathbf{x} = \begin{pmatrix} -4 \\ 13 \end{pmatrix}$$

3. Let V be the set of all 2×2 matrices with real entries, with zero vector, vector addition, and scalar multiplication defined as follows:

   V0) $\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

   V1) $\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} + \begin{pmatrix} y_1 & y_2 \\ y_3 & y_4 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 & x_2 + y_2 \\ x_3 + y_3 & x_4 + y_4 \end{pmatrix}$

   V2) $c\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = \begin{pmatrix} cx_1 & cx_2 \\ cx_3 & cx_4 \end{pmatrix}$

   Prove the distributive property for scalar multiplication over vector addition ($c(\mathbf{x}+\mathbf{y}) = (c\mathbf{x}) + (c\mathbf{y})$ for all $c \in \mathbb{R}$, $\mathbf{x} \in V$, and $\mathbf{y} \in V$). You may use any true properties you know about the real numbers, but no properties of 2×2 matrices other than the ones given above.

4. Prove that if V is a (real) vector space with finitely many elements then V has exactly one element.