Handout #8

Math 2135 Spring 2020

Monday, April 13

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with the following matrix:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Find all real numbers λ such that $A - \lambda I$ is *not* invertible. (Hint: use the determinant.)

2. For each λ you found in the last part of the problem, find all solutions $\mathbf{v}$ for the following equation:

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{v} = \lambda \mathbf{v}$$

(Hint: find the kernel of $A - \lambda I$.)

3. Find a basis $Y = \begin{pmatrix} \mathbf{y}^1 & \mathbf{y}^2 \end{pmatrix}$ of $\mathbb{R}^2$ such that $T(\mathbf{y}^i)$ is a multiple of $\mathbf{y}^i$.

4. Find $[T]_Y^Y$.

5. Find $[T^n]_Y^Y$ for all integers $n \geq 0$, where $T^n = T \circ \cdots \circ T$ (that is, T composed with itself n times).

6. Let $E = \begin{pmatrix} \mathbf{e}^1 & \mathbf{e}^2 \end{pmatrix}$ be the standard basis of $\mathbf{R}^2$. Compute $[\mathrm{id}]_E^Y$ and $[\mathrm{id}]_Y^E$. (Remember: there is a quick formula for the inverse of a 2×2 matrix.)

7. Use change of basis to compute $[T^n]_E^E$ where $E = \begin{pmatrix} \mathbf{e}^1 & \mathbf{e}^2 \end{pmatrix}$ is the standard basis of $\mathbb{R}^2$.

8. Find a formula for the n-th Fibonacci number.