Handout #6

Math 2135 Spring 2020

Friday, March 20

1. Suppose an $m \times n$ matrix A has rank r. Compute the dimensions of:

   (i) $\mathrm{col}(A)$

   (ii) $\mathrm{null}(A)$

   (iii) $\mathrm{col}(A^T)$

   (iv) $\mathrm{null}(A^T)$

2. Let P_n be the vector space of polynomials of degree $\leq n$. Let $D : P_n \to P_n$ be the linear transformation $D(f) = \frac{df}{dx}$.

   (i) What is $\ker(D)$?

   (ii) What is $\mathrm{rank}(D)$?

   (iii) Does D have a left inverse? A right inverse?

   (iv) Why do calculus teachers make you write $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$?

3. Let P_n be the vector space of polynomials of degree $\leq n$. Let $T : P_n \to \mathbb{R}^{n+1}$ be the linear transformation given by the following formula:

$$T(f) = \begin{pmatrix} f(0) \\ f(1) \\ \vdots \\ f(n) \end{pmatrix}$$

   (i) Prove that $\ker T = \{\mathbf{0}\}$.

   (ii) Conclude that T is an isomorphism.

   (iii) Conclude that for any numbers $a_0, \ldots, a_n$ there is a polynomial f such that $f(j) = a_j$ for all $j = 0, \ldots, n$.

   (iv) Find a formula for f.