Handout #5

Math 2135 Spring 2020

Friday, March 13

1. Suppose that A is a 4×4 matrix and let I be the 4×4 identity matrix. Assume that

$$\operatorname{rref}(A|I) = \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 3 & 0 & 0 & 4 & 5 \\ 0 & 0 & 1 & 6 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 0 & 1 & 0 & 9 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 11 & 12 \end{array}\right)$$

   (a) What is $\operatorname{rref}(A)$?

   (b) Find a basis for $\operatorname{null}(A)$. What is its dimension?

   (c) Compute the dimension of $\operatorname{col}(A)$. Do you have enough information to compute a basis for $\operatorname{col}(A)$?

   (d) Does A have a left inverse?

   (e) Does A have a right inverse?

   (f) Find a basis for $\operatorname{null}(A)$.

   (g) Find a matrix Z such that $ZA = \operatorname{rref}(A)$.

   (h) Use the reduced row echelon form above to find a matrix B such that $\operatorname{col}(A) = \operatorname{null}(B)$.

   (i) Find a basis for $\operatorname{col}(A)$.

2. Do Exercise 5.6 of LADW, Chapter 2, §5: Show that the system of vectors

$$\mathbf{v}^1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 5 \\ -3 \end{pmatrix}, \qquad \mathbf{v}^2 = \begin{pmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \qquad \mathbf{v}^3 = \begin{pmatrix} 1 \\ 1 \\ 50 \\ -921 \\ 0 \end{pmatrix}$$

   is linearly independent by completing it to a basis of $\mathbb{R}^5$. Find a matrix B such that $\operatorname{span}\{\mathbf{v}^1, \mathbf{v}^2, \mathbf{v}^3\} = \operatorname{null}(B)$.