

Handout #4

Math 2135 Spring 2020

Friday, March 13

1. For which values of c is the following matrix invertible:

$$\begin{pmatrix} 1 & 2 & 1 \\ 3 & 7 & 3 \\ 2 & 3 & c \end{pmatrix}$$

2. Suppose that A is a 4×4 matrix and let I be the 4×4 identity matrix. Assume that

$$\text{rref}(A|I) = \left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 3 & 0 & 0 & 4 & 5 \\ 0 & 0 & 1 & 6 & 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 0 & 1 & 0 & 9 & 10 \\ 0 & 0 & 0 & 0 & 0 & 1 & 11 & 12 \end{array} \right)$$

- (a) What is $\text{rref}(A)$?
 - (b) Does A have a left inverse?
 - (c) Does A have a right inverse?
 - (d) Find a basis for $\text{null}(A)$.
 - (e) Find a matrix Z such that $ZA = \text{rref}(A)$.
 - (f) Use the reduced row echelon form above to find a matrix B such that $\text{col}(A) = \text{null}(B)$.
 - (g) Find a basis for $\text{col}(A)$.
3. Do Exercise 5.6 of LADW, Chapter 2, §5: Show that the system of vectors

$$\mathbf{v}^1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 5 \\ -3 \end{pmatrix}, \quad \mathbf{v}^2 = \begin{pmatrix} 3 \\ -2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{v}^3 = \begin{pmatrix} 1 \\ 1 \\ 50 \\ -921 \\ 0 \end{pmatrix}$$

is linearly independent by completing it to a basis of \mathbb{R}^5 .