Problem 1. Find all complex numbers x and y that satisfy the following linear equation:

$$(5+3i)x + (-1+2i)y = 1$$

Your solution should take the form $y = ax + b$ where a and b are complex numbers.

Problem 2. Let $\mathbf{v}^1, \ldots, \mathbf{v}^k$ be vectors in $\mathbb{R}^n$. Suppose that a vector equation

$$x_1\mathbf{v}^1 + x_2\mathbf{v}^2 + \cdots + x_k\mathbf{v}^k = \mathbf{w} \tag{$\dagger$}$$

has the following reduced row echelon form:

$$\left(\begin{array}{ccccccc|c} 0 & 1 & 3 & 0 & 0 & a & b & c \\ 0 & 0 & 0 & 1 & 0 & -3 & d & 12 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e \end{array}\right)$$

The letters stand for unknown quantities. In the table below, classify the following statements as true, possible but not necessarily true, or false. Justify your answers briefly below the table.

	True	Possible	False
There are no real numbers $x_1, \ldots, x_k$ satisfying Equation ($\dagger$).			
There is exactly one choice of real numbers $x_1, \ldots, x_k$ satisfying Equation ($\dagger$).			
There is more than one choice of the real numbers $x_1, \ldots, x_k$ satisfying Equation ($\dagger$).			
$\mathbf{w}$ lies in the span of $\mathbf{v}^1, \ldots, \mathbf{v}^k$.			

Problem 3. Prove the following statement:

> *For all scalars r and s in F and for all vectors $\mathbf{v}$ in F^n we have $(r+s).\mathbf{v} = r.\mathbf{v} + s.\mathbf{v}$.*

In your proof, you may use the definitions of vector operations (the zero vector, scalar multiplication, and vector addition), and all true properties of the field F, without justification. Do not use other properties of vector spaces or of F^n in particular without justifying them.

Problem 4. Find all real numbers c such that the vector

$$\begin{pmatrix} -5 \\ c \\ -2 \\ 3 \end{pmatrix}$$

lies in the span of the vectors

$$\begin{pmatrix} 1 \\ -6 \\ -1 \\ -2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ 8 \\ 1 \\ 3 \end{pmatrix}.$$

Problem 5. Let P be the plane in $\mathbb{R}^3$ consisting of those vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that $x + y + z = 0$. Find all vectors in

$$\operatorname{span}\left\{ \begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \right\}.$$

that are contained in P.