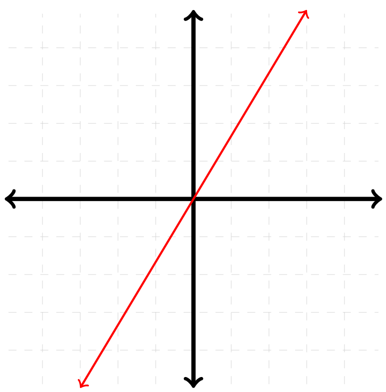
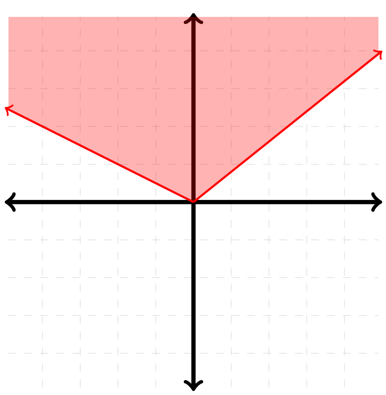
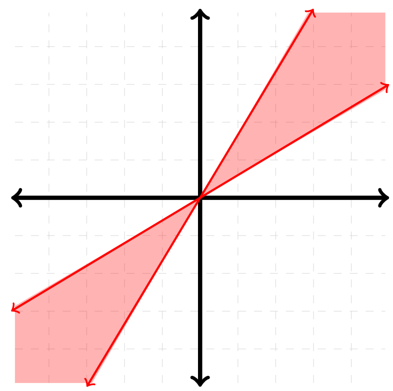
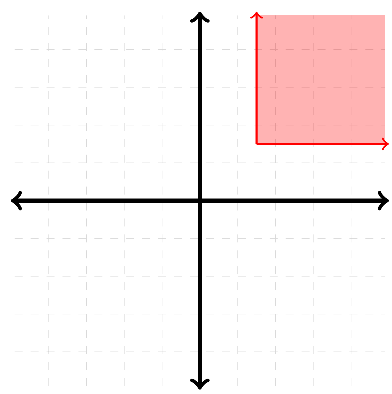
Problem 1. Suppose that $\varphi : \mathbb{R}^4 \to \mathbb{R}$ is known to be a linear functional and that

$$\varphi\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = 14 \qquad \varphi\begin{pmatrix}0\\1\\1\\0\end{pmatrix} = 9 \qquad \varphi\begin{pmatrix}0\\1\\-1\\0\end{pmatrix} = 3$$

Compute the following values of φ:

$$\varphi\begin{pmatrix}1\\0\\0\\1\end{pmatrix} \qquad \text{and} \qquad \varphi\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

Problem 2. Each of the following describes, either with symbols or a picture, a subset W of $\mathbb{R}^2$. Which of these subsets of $\mathbb{R}^2$ are subspaces? Justify your answers by indicating which properties of a subspace each possesses (you may use the notation from class: a), $*$), b), c)).

1. $\mathbb{R}^2$
2. 
3. 
4. $\varnothing$
5. 
6. 

Problem 3. Find a nonzero row vector $\vec{a}$ in $\mathbb{R}_4$ such that $\ker(\vec{a})$ contains all of the following three vectors:

$$\vec{u}^1 = \begin{pmatrix}3\\4\\2\\-1\end{pmatrix} \qquad \vec{u}^2 = \begin{pmatrix}-6\\0\\4\\1\end{pmatrix} \qquad \vec{u}^3 = \begin{pmatrix}8\\2\\0\\-2\end{pmatrix}$$

Problem 4. Let P be the vector space of all polynomials with real coefficients. Define $I : P \to \mathbb{R}$ by the following formula:

$$I(\vec{f}) = \int_{-1}^{1} \vec{f}(x)\, dx$$

Prove that I is a linear functional.

Problem 5. Let V be set of all functions from $\mathbb{R}$ to $\mathbb{R}$. As discussed in class, V is a vector space over the field $\mathbb{R}$ (you do not need to prove this). Let W be the subset of V consisting of all *nondecreasing* functions. (Recall that f is nondecreasing if $x \leq y$ implies $f(x) \leq f(y)$.) Is W a subspace of V? Justify your answer fully by providing a proof or counterexample to each of the properties required of a subspace.

Problem 6. Let V be a vector space over the field F. Let W be the vector space of all functions from V to F. Let V^* be the set of all linear functionals on V. Prove that V^* is a subspace of W.