

$A$  is an  $m \times n$  matrix and  $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is the linear transformation  $T(\mathbf{x}) = A\mathbf{x}$ . The two columns show equivalent properties of  $A$  and  $T$ :

There is a $n \times m$ matrix $C$ such that $CA = I_n$	There is a $n \times m$ matrix $D$ such that $AD = I_m$
$\text{Nul}(C) = \{\mathbf{0}\}$	$\text{Col}(C) = \mathbb{R}^m$
$T$ is one-to-one	$T$ is onto
$\text{rref}(A)$ has a pivot in every column	$\text{rref}(A)$ has a pivot in every row
$\text{rref}(A)$ has $n$ pivots	$\text{rref}(A)$ has $m$ pivots
$\dim \text{Nul}(A) = 0$	$\dim \text{Col}(A) = m$
$\text{rank}(A) = n$	$\text{rank}(A) = m$
the only solution to $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$	for every $\mathbf{b}$ in $\mathbb{R}^m$ , the equation $A\mathbf{x} = \mathbf{b}$ has a solution $\mathbf{x}$
the columns of $A$ are linearly independent	the columns of $A$ span $\mathbb{R}^m$
$A$ has an invertible $n \times n$ submatrix	$A$ has an invertible $m \times m$ submatrix
$A$ has a nonzero $n \times n$ minor	$A$ has a nonzero $m \times m$ minor

Some study advice: make sure you understand why all of the properties in each column are equivalent.