Graded Problem Set 3

Math 2130 — Fall 2022

due Friday, 2 December

Instructions: You may use our textbook and any other linear algebra textbooks you like. You may also watch previously recorded linear algebra lectures. You may discuss the problems with other members of the class and with your professor. You may not discuss (either aloud or electronically) the problems with anyone outside of the class. In particular, do not ask for help at the MARC, do not discuss the problems with tutors, and do not solicit help online — all of these will be considered cheating.

Once you are satisfied with your solution to a problem, you should write up your solution **without consulting any resources** except your own understanding of linear algebra. This includes any notes you might have taken while consulting resources earlier.

Keep track of all resources you consult and cite them. This includes any discussions about the problems you have with other students or with your professor and any texts or websites you consult. Citations should be a separate page of your submission and should specific enough to be checked: give page numbers of books and give specific URLs of web resources. You should include a references section even if you do not consult any sources. Submissions without a references section will not be graded.

Submit your solutions on Canvas: https://canvas.colorado.edu/courses/86501/assignments/1526061. Do **not** put your name on your submission (this allows me to grade your work anonymously).

1. Let A be the matrix $\begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix}$.

   (a) Compute the eigenvectors and eigenvalues of A.

   (b) Compute A^{2022}.

   (c) There are two possibilities for $\lim_{n\to\infty} \frac{A^n\vec{v}}{\|A^n\vec{v}\|}$ when $\vec{v}$ is *not* an eigenvector of A. (In other words, which direction can the vectors $A^n\vec{v}$ approach as n becomes large?)

2. Let $\vec{u} = \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ and let $\vec{v} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$. Compute the eigenvectors and eigenvalues of the 3×3 matrix $A = \vec{u}\vec{v}^T$.

3. Suppose that A and B are square matrices and that $\vec{x}$ is an eigenvector of A with eigenvalue 3 and an eigenvector of B with eigenvalue 5. Explain why $\vec{x}$ is also an eigenvector of $A^4 + ABA^2$ and compute its eigenvalue.

4. Let W be the plane in $\mathbb{R}^3$ spanned by the vectors $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$. Let A be the matrix that reflects vectors across W.

   (a) Find a nonzero vector $\vec{x}$ that is perpendicular to W.

   (b) Find the eigenvalues and eigenvectors of A.

   (c) (Optional) Compute the matrix of A using the projection onto the plane W or using the projection onto the line $W^\perp$.

5. For which values of c is $A = \begin{pmatrix} 3 & 2 \\ c & 4 \end{pmatrix}$

(i) diagonalizable by a real change of coordinates;

(ii) diagonalizable by a non-real complex change of coordinates;

(iii) not diagonalizable?