Graded Problem Set 2

Math 2130 — Fall 2022

due Friday, October 7

Instructions: You may use our textbook and any other linear algebra textbooks you like. You may also watch previously recorded linear algebra lectures. You may discuss the problems with other members of the class and with your professor. You may not discuss (either aloud or electronically) the problems with anyone outside of the class. In particular, do not ask for help at the MARC, do not discuss the problems with tutors, and do not solicit help online — all of these will be considered cheating.

Your do your final writeup without consulting any sources except for the online row reduction tool at http://math.colorado.edu/~jonathan.wise/math2130/ps2.html. Once you are satisfied with your solution to a problem, you should write up your solution **without consulting any resources** except your own understanding of linear algebra. This includes any notes you might have taken while consulting resources earlier.

Keep track of all resources you consult and cite them. This includes any discussions about the problems you have with other students or with your professor and any texts or websites you consult. Citations should be a separate page of your submission and should specific enough to be checked: give page numbers of books and give specific URLs of web resources. You should include a references section even if you do not consult any sources. Submissions without a references section will not be graded.

Submit your solutions on Canvas: https://canvas.colorado.edu/courses/86501/assignments/1500477. Do **not** put your name on your submission (this allows me to grade your work anonymously).

1. Let

$$A = \begin{pmatrix} 2 & 17 & 13 & 71 & 77 \\ 11 & 6 & -16 & -47 & 50 \\ -5 & -22 & -12 & -75 & -105 \\ 2 & 4 & 0 & 6 & 25 \end{pmatrix}$$

   (a) Use `http://math.colorado.edu/~jonathan.wise/math2130/ps2.html` to compute the reduced row echelon form of A. In your answer, give the reduced row echelon form and the sequence of commands you used to find it. Please copy and paste the commands from your browser window (do not hand write them) to make it easier for me to check them.

   (b) Find a matrix B such that $BA = \operatorname{rref}(A)$. You may use the computer again for this part.

   (c) Find a basis for the column space of A.

   (d) Find a basis for the null space of A.

   (e) Find a matrix V such that $N(A) = C(V)$.

   (f) Find a matrix U such that $C(A) = N(U)$.

   (g) Find the dimensions of the row space and left null space of A.

   (h) For which numbers b does $A\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ b \end{pmatrix}$ have a solution?

   (i) Find the general solution to $A\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ b \end{pmatrix}$ when b satisfies your condition from the last part.

2. Remember that a subspace of $\mathbb{R}^n$ is a set V of vectors with the following properties:

   V1 $\vec{0}$ is in V,

   V2 if c is a scalar and $\vec{v}$ is in $\vec{V}$ then $c\vec{v}$ is in V.

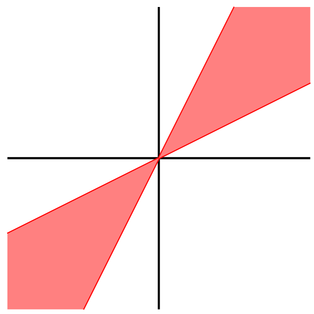
   V3 if $\vec{v}$ and $\vec{w}$ is in V then $\vec{v} + \vec{w}$ is in V.

None of the following examples is a subspace of $\mathbb{R}^2$. Explain why not in each case.

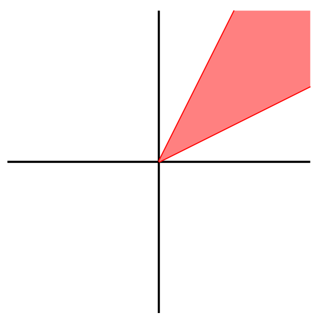
(a) All vectors in $\mathbb{R}^2$ whose coordinates are integers.

(b) All solutions $\begin{pmatrix} x \\ y \end{pmatrix}$ to the equation $x + 2y = 3$.

(c) The shaded area in red:



(d) The shaded area in red:



3. Find a 4×4 matrix A whose column space is spanned by the vectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 7 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2 \\ 0 \\ 2 \\ 2 \end{pmatrix}$$

and whose null space is spanned by the vectors

$$\begin{pmatrix} 3 \\ -1 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ 5 \\ 1 \\ 1 \end{pmatrix}.$$

There are multiple correct approaches and multiple correct answers to this problem, so make sure to show your process clearly.

4. A 4×4 matrix A and a vector $\vec{b} \in \mathbb{R}^4$ are unknown. The set of solutions $\vec{x}$ to $A\vec{x} = \vec{b}$ contains a collection of 3 *linearly independent* vectors. What are the possibilities for $\dim N(A)$ and $\dim C(A)$?