

# Problem Set 1 — Suggested revision problems

Math 2130 — Fall 2022

September 19, 2022

The following problems have been suggested by your classmates for revision of Problem Set 1. You may wish to use some of them for your own revision, but you are not required to.

1. Compute the following matrix-vector product:

$$\begin{pmatrix} -3 & 2 & 5 & 0 \\ 0 & 4 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 6 \\ 10 \end{pmatrix}$$

2. (Mara Backsen) Suppose that  $\vec{u}$ ,  $\vec{v}$ ,  $\vec{w}$ , and  $\vec{x}$  are vectors in  $\mathbb{R}^4$ . Explain whether these vectors are either definitely linearly independent, possibly linearly independent or dependent, or definitely linearly dependent.
3. There are unknown vectors  $\vec{u}$ ,  $\vec{v}$ ,  $\vec{w}$ , and  $\vec{x}$  in  $\mathbb{R}^3$  with the following dot products:

$$\begin{array}{lll} \vec{u} \cdot \vec{u} = 1 & \vec{v} \cdot \vec{v} = 1 & \vec{w} \cdot \vec{w} = 1 \\ \vec{u} \cdot \vec{v} = \frac{1}{2} & \vec{u} \cdot \vec{w} = \frac{1}{2} & \vec{v} \cdot \vec{w} = \frac{1}{2} \\ \vec{x} \cdot \vec{u} = 0 & \vec{x} \cdot \vec{v} = 0 & \vec{x} \cdot \vec{x} = 4 \end{array}$$

Determine all possible values for  $\vec{x} \cdot \vec{w}$ .

4. Suppose that  $\vec{u}$ ,  $\vec{v}$ ,  $\vec{w}$ ,  $\vec{b}$ , and  $\vec{c}$  are vectors in  $\mathbb{R}^3$  and  $A$  is the matrix  $\begin{pmatrix} \vec{u} & \vec{v} & \vec{w} \end{pmatrix}$ . If the equation  $A\vec{x} = \vec{b}$  has more than one solution, how many solutions can there be to  $A\vec{x} = \vec{c}$ ?
5. (Boston College, care of Nico Rodriguez) Find the values of  $h$  for which the following vectors are linearly independent:

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix} h \\ 1 \\ -h \end{pmatrix} \quad \vec{v}_3 = \begin{pmatrix} 1 \\ 2h \\ 3h+1 \end{pmatrix}$$