# Graded Problem Set 1

Math 2130 — Fall 2022

due Friday, September 2

*Instructions:* You may use our textbook and any other linear algebra textbooks you like. You may also watch previously recorded linear algebra lectures. You may discuss the problems with other members of the class and with your professor. You may not discuss (either aloud or electronically) the problems with anyone outside of the class. In particular, do not ask for help at the MARC, do not discuss the problems with tutors, and do not solicit help online — all of these will be considered cheating.

**You should not write anything that will be submitted while consulting outside sources.** Once you are satisfied with your solution to a problem, you should write up your solution **without consulting any resources** except your own understanding of linear algebra. This includes any notes you might have taken while consulting resources earlier.

Keep track of all resources you consult and cite them. This includes any discussions about the problems you have with other students or with your professor and any texts or websites you consult. Citations should be specific enough to be checked: give page numbers of books and give specific URLs of web resources.

Writeup: Do not consult any sources or notes from your investigation phase.

1. Suppose that $A$ is an $m \times n$ matrix and that $\vec{x}$ and $\vec{y}$ are vectors such that $A\vec{x} = \vec{y}$. How many rows do $\vec{x}$ and $\vec{y}$ each have?

2. Find a matrix $A$ and vectors $\vec{x}$ and $\vec{b}$ that allow you to rewrite the

system of linear equations

$$\begin{aligned} 8x + 4y + 2w &= 1 \\ y + z &= 0 \\ 2x + w &= -1 \end{aligned}$$

as a vector equation $A\vec{x} = \vec{b}$. The entries of $\vec{x}$ should all be variables and the entires of $A$ and $\vec{b}$ should all be real numbers.

3. Find all unit vectors perpendicular to $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

4. Suppose that $\vec{v}$ and $\vec{w}$ are unknown vectors in $\mathbb{R}^n$ that make an angle of $\frac{\pi}{3}$. Assume that $\|\vec{v}\| = 5$ and $\|\vec{w}\| = 6$. Compute $\|\vec{v} + \vec{w}\|$.

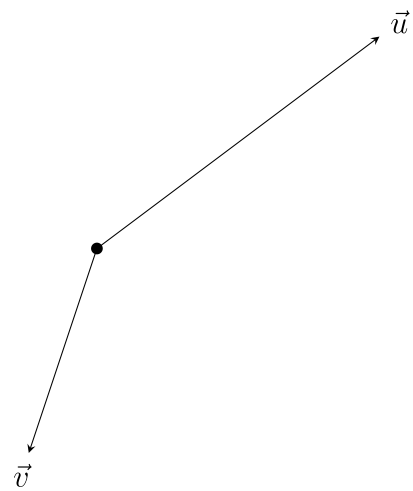
5. Do the following sets of vectors generate a line, a plane, or all of $\mathbb{R}^3$? Give a very brief explanation of how you know in each case.

   (a) $\vec{u} = \begin{pmatrix} 4 \\ -2 \\ 6 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$

   (b) $\vec{u} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$, and $\vec{w} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

   (c) $\vec{u} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, and $\vec{w} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

6. The following image shows 2 vectors, $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^2$. Shade the region containing all vectors $\vec{x}$ such that $\vec{x} \cdot \vec{u} < 0$ and $\vec{x} \cdot \vec{v} = 0$.




7. Suppose that $\vec{u}$, $\vec{v}$, $\vec{w}$, and $\vec{x}$ are all vectors in $\mathbb{R}^3$ and that we know the following about their dot products:

$$\begin{array}{cccc} \vec{u}\cdot\vec{v}=0 & \vec{u}\cdot\vec{x}=0 & \vec{v}\cdot\vec{x}=0 & \vec{w}\cdot\vec{x}=0 \\ \vec{u}\cdot\vec{u}=1 & \vec{v}\cdot\vec{v}=1 & \vec{w}\cdot\vec{w}=1 & \vec{x}\cdot\vec{x}=1 \end{array}$$

   (a) Is $\vec{x}$ a linear combination of $\vec{u}$, $\vec{v}$, and $\vec{w}$? Why or why not?

   (b) Is $\vec{w}$ a linear combination of $\vec{u}$, $\vec{v}$, and $\vec{x}$? Why or why not?

8. Suppose that $\vec{u}$, $\vec{v}$, and $\vec{w}$ are unknown vectors that all lie in a plane. Predict how many vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ will solve the following equation:

$$x\vec{u}+y\vec{v}+z\vec{w}=\vec{0}$$

   Your answer should either be that there are no solutions, there is exactly one solution, or there is more than one solution. Explain how you know.